1. Chapter 1, Sections 1.3.2 and 1.3.3, pp. 8 – 11. Dimensional Analysis and Unit Law

1.1 Use Table 1.5 to check which of the following equations is dimensionally valid and describes the Lorentz's force equation correctly

a) $\boldsymbol{F}_{em} = Q_e(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})$ b) $\boldsymbol{F}_{em} = Q_e(\boldsymbol{D} + \boldsymbol{v} \times \boldsymbol{H})$

c) $\boldsymbol{F}_{em} = Q_m(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})$ d) $\boldsymbol{F}_{em} = Q_e\boldsymbol{E} + Q_m\boldsymbol{v} \times \boldsymbol{B}$

1.2 Use Table 1.5 to check which of the following equations is dimensionally valid and describes the 1st Maxwell's equation in the differential form correctly

a) $-\boldsymbol{\nabla} \times \boldsymbol{E} = \frac{\partial \boldsymbol{H}}{\partial t} + \boldsymbol{j}_{mV}$ c) $-\boldsymbol{\nabla} \times \boldsymbol{E} = \frac{\partial \boldsymbol{B}}{\partial t} + \boldsymbol{j}_{eV}$

b) $-\boldsymbol{\nabla} \times \boldsymbol{E} = \frac{\partial \boldsymbol{B}}{\partial t} + \boldsymbol{j}_{mV}$ d) $-\boldsymbol{\nabla} \times \boldsymbol{D} = \frac{\partial \boldsymbol{H}}{\partial t} + \boldsymbol{j}_{mV}$

1.3 Use Table 1.5 to check which of the following equations is dimensionally valid and describes the 2nd Maxwell's equation in the differential form correctly

a) $\boldsymbol{\nabla} \times \boldsymbol{H} = \frac{\partial \boldsymbol{E}}{\partial t} + \boldsymbol{j}_{eV}$ c) $\boldsymbol{\nabla} \times \boldsymbol{H} = \frac{\partial \boldsymbol{D}}{\partial t} + \boldsymbol{j}_{mV}$

b) $\boldsymbol{\nabla} \times \boldsymbol{H} = \frac{\partial \boldsymbol{D}}{\partial t} + \boldsymbol{j}_{eV}$ d) $\boldsymbol{\nabla} \times \boldsymbol{B} = \frac{\partial \boldsymbol{D}}{\partial t} + \boldsymbol{j}_{eV}$

1.4 Use Table 1.5 to check which of the following equations is dimensionally valid and describes the *electric* charge conservation law correctly

a) $\oiint_A \boldsymbol{j}_{mV} \circ d\boldsymbol{A} + \frac{dQ_e(t)}{dt} = 0$ c) $\oiint_A \boldsymbol{j}_{eV} \circ d\boldsymbol{A} + \frac{dQ_m(t)}{dt} = 0$

b) $\oint_L \boldsymbol{j}_{eV} \circ d\boldsymbol{A} + \frac{dQ_e(t)}{dt} = 0$ d) $\oiint_A \boldsymbol{j}_{eV} \circ d\boldsymbol{A} + \frac{dQ_e(t)}{dt} = 0$

1.5 The vector $\boldsymbol{H}$ of *static* magnetic field is measured in base units [A/m] along with some closed path *L*. Use Table 1.5 to check which of the following equations is dimensionally valid and describes the Ampere's law correctly

a) $\oint_L \boldsymbol{H} \circ d\boldsymbol{L} = I_m$ c) $\oint_L \boldsymbol{H} \circ d\boldsymbol{L} = Q_e$

b) $\oint_L \boldsymbol{H} \circ d\boldsymbol{L} = I_e$ d) $\oint_L \boldsymbol{H} \circ d\boldsymbol{L} = Q_m$

1.6 The vector $\boldsymbol{D}$ of electric displacement field is measured in base units [As/m^2] along some closed surface *A*. Use Table 1.5 to check which of the following equations is dimensionally valid and describes the 3rd Maxwell's equation (Gauss's law) correctly

a) $\oiint_A \boldsymbol{D} \circ d\boldsymbol{A} = I_e$ c) $\oiint_A \boldsymbol{D} \circ d\boldsymbol{A} = I_m$

b) $\oiint_A \boldsymbol{D} \circ d\boldsymbol{A} = Q_e$ d) $\oiint_A \boldsymbol{D} \circ d\boldsymbol{A} = Q_m$

1.7 The vector $\boldsymbol{B}$ of magnetic inductance field is measured in base units [As/m^2] on some closed surface *A*. Use Table 1.5 to check which of the following equations is dimensionally valid and describes the 3rd Maxwell's equation (Gauss's law) correctly

a) $\oiint_A \boldsymbol{D} \circ d\boldsymbol{A} = I_e$　　c) $\oint_L \boldsymbol{D} \circ d\boldsymbol{L} = I_e$

b) $\oiint_A \boldsymbol{D} \circ d\boldsymbol{A} = Q_e$　　d) $\oiint_A \boldsymbol{D} \circ d\boldsymbol{A} = Q_m$

1.8 You found reading several publications that power P_Σ emitted by the infinitesimal current element is defined as

a) $P_\Sigma = \frac{8\pi}{3}\left(\frac{I_e^{exc}\Delta l}{2\sqrt{2}\lambda}\right)^2 / Z_0$　　c) $P_\Sigma = \frac{8\pi}{3}\left(\frac{I_e^{exc}\Delta l}{2\sqrt{2}\lambda}\right)^2 Z_0$

b) $P_\Sigma = \frac{8\pi}{3}\left(\frac{I_e^{exc}\Delta l}{2\sqrt{2}\lambda}\right) Z_0^2$　　d) $P_\Sigma = \frac{8\pi}{3}\left(\frac{I_e^{exc}\Delta l}{2\sqrt{2}\lambda}\right)^2 Z_0^2$

Which expression may be correct if Δl is the physical length of the element, I_e^{exc} is the electric current, λ is a wavelength in [m], and Z_0 is the impedance of free space in [Ohms] (check Section 4.2 in Chapter 4 for details)?

1.9 An inductance L [H] is attached in parallel or series to a capacitance C [F]. Using data from Table 1.5, find such combination of these values that correctly defines the angular resonance frequency ω [1/s] of such parallel resonance circuit.

1.10 The distributed parameters of some transmission line are $\mathcal{L}'$ [H/m] and C' [F/m] (see Sections 6.1 and 6.5 of Chapter 6 for details). Using data from Table 1.5, find such combination of these parameters that correctly defines the characteristic line impedance Z_c [Ohms].

2. Chapter 1, Sections 1.6.1 - 1.6.8, pp. 19 – 26. Electric and Magnetic Field Vectors

2.1 What is the strength and direction of the static and uniform electric field ***E*** [V/m] (use expression (1.17)) pushing an electron of 1.60217662×10^{-19} [C] in the positive direction of *z*-axis by the force 1×10^{-15} [N]? Compare this field strength with the air breakdown strength 3×10^{6} [V/m]. Might we watch light spike in air or corona as multiple electrons are accelerated simultaneously? *Hint.* Search Wikipedia.

2.2 Using data from the problem 2.1 and Newton's second law of motion $\boldsymbol{F} = m\boldsymbol{a}$, find the electron acceleration of $m = 9.10938356\times10^{-31}$ [kg], its velocity, and traveled distance for a time period of 10^{-8} [s]. Compare this velocity with speed of light c = 299 792 458 [m/s]. Is it possible for such accelerating electron to reach the speed of light? If your answer is yes, how long does it take?

2.3 Using data from the problem 2.1, find the energy W_e [J] (use the expression (1.19)) that the electric field transfers into the kinetic energy of accelerating an electron. Calculate the kinetic energy of the electron as $W_k = mv^2/2$ and check that $W_k = W_e$. Recalculate W_e in the electron-volt [eV] assuming that 1 [eV] = 1.60×10^{-19} [J].

2.3 Using data from the problem 2.1 - 2.3 find the electric potential U_e [V] (use expression (1.21)) that accelerated the electron.

2.4 Execute the Matlab script below into Matlab Command Window and check your calculations.

```
clc; close all; clear; e = 1.60217662e-19; m = 9.10938356e-31; c = 299792458; F=1e-13; t=1e-9;
E=F/e;a=F/m;v=a*t;ratio=v/c;dist=a*t^2/2;We=e*E*dist;Wk=m*v^2/2;WeV=We/1.6e-19;Ue=We/e;
X1 = ['|E| =',num2str(E),' V/m;','    a = ',num2str(a),' m/s^2;','   v = ',num2str(v),' m/s;'];disp(X1);
X2 = ['v/c = ',num2str(ratio),';','    dist = ',num2str(dist),' m;','    We = ',num2str(We),' J;'];disp(X2);
X3 = ['Wk = ',num2str(Wk),' J;','   WeV = ',num2str(WeV),' eV;','    Ue = ',num2str(Ue),' V;'];disp(X3);
```

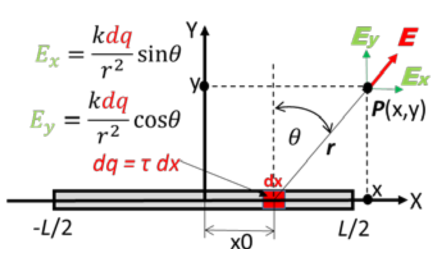
2.5 The script below calculates and plots 2D equipotential lines of electric potential Ue and lines of E-force (red curve with arrows) produced by any number of static point charges located inside the square -4<= x <=4, -4<= y <=4 [m] .

```
clc; close all; clear
figure('units','normalized','outerposition',[0 0 1 1]); hax=axes;grid minor; hold all
k = 8.987E9;
display('All charges must be put inside area -4<=x<=4,-4<=y<=4 [m]')
n=input('enter total number of charges: ');
for jj=1:n
   Q(jj)=input(['enter charge Q(',num2str(jj), ') value in [C]: ']);
   x(jj)=input(['enter charge Q(',num2str(jj),') x-coordinate in [m]: ']);
   y(jj)=input(['enter charge Q(',num2str(jj),')  y-coordinate in [m]: ']); end
[X,Y] = meshgrid(-4:0.02:4); Ue = zeros(size(X));
for ii = 1:numel(Q)
   Ue = Ue + k * Q(ii) ./ hypot(x(ii)-X, y(ii)-Y); end
[Ex,Ey] = gradient(Ue,0.02,0.02); hContour = contour(X,Y,Ue,201); hColorbar = colorbar;
ylabel(hColorbar,'Electric potential (Ue)','FontWeight','Bold'); validColumns = all(isfinite(Ex) & isfinite(Ey));
hLines = streamslice(X(:,validColumns),Y(:,validColumns),-Ex(:,validColumns),-Ey(:,validColumns));
set(hLines,'Color','r'); xlabel('-4 \leq X \leq 4','FontWeight','bold'); ylabel('-4 \leq Y \leq 4','FontWeight','bold')
```

Copy and paste it into Matlab Command Window. Display data for single charge {Q(1)=1, x=0, y=0}, dipole {Q(1)=1, x=0, y=1; Q(2)=±1, x=0, y= -1} and quadrupole (four charges in square corners). Explain how to estimate the relative strength and direction of E-field (see Section 1.6.3) using this plot. Show that the vector of E-field from point charges can be obtained by taking the vector sum of the E-fields of the individual charges.

2.6 Type http://www.falstad.com/vector3de/ in your browser and press Enter. Click "Full screen version" on the bottom of the opened page. In the "Field selection" on the right top corner choose "dipole" and "Display: Field Lines or Equipotentials." Go to "Show Z Slice" and compare with the plots you have received solving problem 2.5. Check different combinations of charges and material bodies. Explain images. Play and enjoy.

2.7 Suppose a charge Q spreads uniformly along the x-axis with linear density $\tau = Q/L$ where L is the total length of the charged line. E-field due to the total charge Q can be calculated by superposing (really integrating) the E-field of infinitesimal charge dq using the equation (1.24) in Chapter 1. The trigonometric functions are equal $\sin\theta = (x - x0)/r$ and $\cos\theta = y/r$ where $r^2 = (x - x0)^2 + y^2$. Allowing $x0$ vary from $-L/2$ to $L/2$

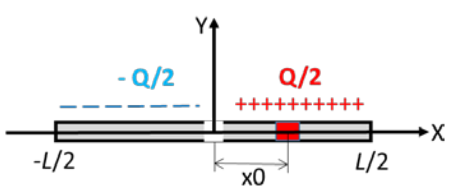


write down the integral for each component of E-field and verify them reading the Matlab script below. Then copy and paste it into Matlab Command Window. Check two special cases of the short line (L = 0.1m, for example) and relatively long line of L = 9m. Compare plots and explain the differences in E-field structures. What kind of E-field structure could you expect in case of an infinitely long line, i.e. $L \to \infty$? Check the case of a negative charge.

```
clc; close all; clear
figure('units','normalized','outerposition',[0 0 1 1]);
hax=axes; grid minor; hold all
k = 8.987E9;a=5;
display('      Length L of finit line of charge L<=9m')
L=input('Enter the length L of finit line of charge in [m]: ');
tau=input('Enter the charge density of charges in [C/m]: ');
[X,Y] = meshgrid(-a:0.05:a); x0=linspace(-L/2,L/2,101);
for jj=1:size(x0,2)
   r(jj,:,:)=sqrt((x0(jj)-X).^2+Y.^2); x1(jj,:,:)=X-x0(jj);
   dEx(jj,:,:)=k*tau*x1(jj,:,:)./r(jj,:,:).^3; dEy(jj,:,:)=k*tau./r(jj,:,:).^3;
end
Ex=squeeze(trapz(dEx,1)); Ey=Y.*squeeze(trapz(dEy,1));
hLines = streamslice(X,Y,Ex,Ey,3); set(hLines,'Color','r');
xlabel('-5 \leq X \leq 5','FontWeight','bold');
ylabel('-5 \leq Y \leq 5','FontWeight','bold'); plot(x0,0*x0,'k*'); axis tight
```

If you like a challenge present all integrals in closed form and then replace the Matlab function 'trapz' performing the Trapezoidal numerical integration.

2.8 Assume that the charge line comprises two separate halves carrying equal but opposite charges $\pm Q/2$. Copy and paste the script below into Matlab Command Window. Explain how to estimate the relative strength and direction of E-field (see Section 1.6.3) using this plot. Mark area with the highest intensity of E-field.




```
clc; close all; clear
figure('units','normalized','outerposition',[0 0 1 1]); hax=axes; grid minor; hold all
k = 8.987E9; a=5;
display('      Length L of finite line of charge L<=9m')
L=input('Enter the length L of finite line of charge in [m]: ');
tau=input('Enter the charge linear density of charges in [C/m]: ');
[X,Y] = meshgrid(-a:0.05:a); x0=linspace(-L/2,L/2,200);
for jj=1:size(x0,2)
   r(jj,:,:)=sqrt((x0(jj)-X).^2+Y.^2); x1(jj,:,:)=X-x0(jj);
   if x0(jj)>=0
     dEx(jj,:,:)=k*tau*x1(jj,:,:)./r(jj,:,:).^3; dEy(jj,:,:)=k*tau./r(jj,:,:).^3;
   else
     dEx(jj,:,:)=-k*tau*x1(jj,:,:)./r(jj,:,:).^3; dEy(jj,:,:)=-k*tau./r(jj,:,:).^3;
   end
end
Ex=squeeze(trapz(dEx,1)); Ey=Y.*squeeze(trapz(dEy,1));
hLines = streamslice(X,Y,Ex,Ey,3); set(hLines,'Color','r'); xlabel('-5 \leq X \leq 5','FontWeight','bold');
ylabel('-5 \leq Y \leq 5','FontWeight','bold'); plot(x0,0*x0,'k*'); axis tight
```

Explain the results and compare them with the near-field E-field structure around an elementary electric dipole depicted in Figure 4.3.1 of Chapter 4. If you like a challenge present all integrals in closed form. Then demonstrate analytically that the dominant components of dipole E-near-fields (see (5.20) and (5.21) in Chapter 5) and this charged line are almost identical and both deviate as $\sim 1/r^2$ and $\sim 1/r^3$ nearby. Show that the static and certainly nonradiating charged line is not able to create the E-far-field component $\sim 1/r$.