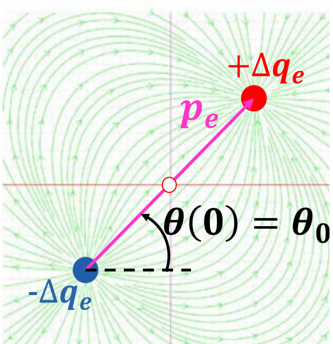
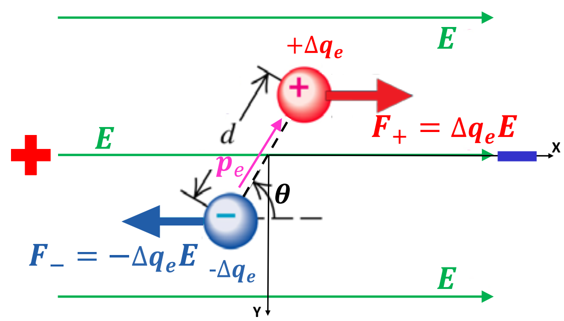
To boost the flavor of electromagnetics, we accompanied many problems with short Matlab code acting as *a calculator and result illustrator*. We completely ignored the description of algorithms behind the codes because they are often quite complicated and required ample, very sophisticated as well abstract mathematical specifics poorly helping to understand the physical picture. Such approach let us shift the focus from the often emasculated and practically fruitless problems primarily based on math transformations to more complicated but close to practical tasks. We hope that it helps to visualize typically invisible EM fields and stimulate in-depth analysis and following discussion of the fundamental principles. Projecting images onto a big screen, the lectures may organize whole class conversation making the audience to be significantly engaged.
We cheer our readers to look through Matlab scripts to discover and sophisticate the algorithm embedded in them. We encourage to use the student edition of MATLAB and CST STUDIO SUITE® to get enhanced problem understanding. Sorry for compact transcription of scripts that is dictated by the blog environment.
**A better readable version of scripts will be sent to you upon request left on the Contact page.**

**Attention. Regrettably, copy and paste into Matlab Command Window saves appropriate Matlab format just in Chrome Web Browser. You have to restart [https://emfieldbook.com/](https://emfieldbook.com/) in [https://www.google.com/](https://www.google.com/) if your browser is different.**

## Section 2.1. Torque Exerted by Electric and Magnetic Field

2.1 **Rotation of Ball-and-Stick Model of Electric Dipole in Uniform E-field**. Suppose a monopole duo similar to a field sensor #2 (see Section 1.4.1 in Chapter 1) is placed in a uniform E-field at the moment $t = 0$, and it is motionless, i.e. $(d\theta/dt)_{t=0} = 0$. The dipole initial angular displacement is $\theta(0) = \theta_0$. Figure on the left depicts the E-force lines around this dipole at $t = 0$. Figure on the right demonstrates the Lorentz's forces $\boldsymbol{F}_{\pm}$ (check expression (1.11)) are exerted by the applied E-field. Eventually, the dipole is forced spinning as a solid assembly around its center. The source of the external E-field is not shown except the polarity of far-away charges (large red sign + and blue -) that generate this field. The following Matlab script is built around the exact solution[1] of the equation of motion. For simplification, it was assumed that the monopole duo consists of $\Delta q_e = \pm e$ with mass $m = m_e$ (see Table 1.5) and separation $d = 6*10^{-11}$m. The reader may edit the 5th line of acript if desired.




Copy, paste, and run the script into Matlab Command Window starting from the relative small angular displacements $\leq 60°$ 60 and the dissipation factor equal to 0. Figure 1 plots the exact $y$-axis displacement $y = \sin(\theta(t))$, bold blue line, and the same displacement assuming the dipole pure harmonic angular oscillation (red asterisk line) with frequency $f_0 = \sqrt{2eE/(md)}$ (recollect the motion of a simple gravity pendulum from course of mechanics). Figure 2 is the animation illustrating the dipole angular movement where the black vector depict the value (not in scale) and direction of the torque vector $\boldsymbol{T}$ (read Section 2.1 and expression (2.7) in Chapter 2) as a function of time. The magenta vector is the direction of dipole moment $\boldsymbol{p}_e$.

[1] A. Beléndez1, C. Pascual, D.I. Méndez, T. Beléndez, C. Neipp, Exact solution for the nonlinear pendulum, Sociedade Brasileira de Física, 2007, [http://www.scielo.br/scielo.php?script=sci_arttext&pid=S1806-11172007000400024](http://www.scielo.br/scielo.php?script=sci_arttext&pid=S1806-11172007000400024)

```
clc; close all; clear
E=input('Enter the intensity of uniform E-field [V/m] = ');
dis=input('Enter the relative dissipation factor between 0 (no loss) and 1 (high loss) = ');
theta0=input('Enter the initial angular displacement > 0 or < 180[degrees] = ');
m=9.10938356e-31; e=1.60217662e-19; d=6e-11; omega0=sqrt(2*e*E/(m*d));
NN=2.4e2; theta0=pi*theta0/180; t=linspace(0,8*pi/omega0,NN);
ss=sin(theta0/2); k=ss^2; K=ellipke(k); u=K-omega0*t;
SN=jacobiSN(u,k); Arg=SN*ss; omega=pi*omega0/(2*K);
Theta=2*asin(Arg).*exp(-omega*t*dis); Th=1;
f1=figure('units','normalized','outerposition',[.2 .2 .6 .6]); axis square
movegui(f1,'west'); plot(t,sin(Theta),'LineWidth',2); grid minor; hold on; plot(t,0*Theta,'-');
plot(t,sin(omega*t+pi/2).*sin(theta0),'*r');
axis tight; xlabel('\bf Time [s]'); ylabel('\bfTorque (Kinetic Energy of Rotation) T(t) [J/radian]');
 title(['\bf','Initial Angular Displacment = ',num2str(theta0*180/pi),'deg'])
text(t(end/4),sin(theta0)-0.1,['\bfOscillation Frequency f_0 = ',num2str(omega0/(2*pi)/1e9),' [GHz]'])
f2=figure;  movegui(f2,'east'); grid minor; xlabel('\bfX-axis'); ylabel('\bfY-axis');
steps=length(t); axis([-1 1 -1 1 -Th/2 Th/2]*2); hold on;
hp=patch([1 -1 -1 1]*2, [1 1 -1 -1]*2, [0 0 0 0]); alpha(hp,0.2)
hPlot = plot(NaN,NaN,'--b','LineWidth',2); h3Plot = plot(NaN,NaN,'--b','LineWidth',2); plot(0,0,'or'); r12=1;
hq1=quiver3(-2,0,0,4,0,0,'g'); set(hq1,'LineWidth',3,'AutoScale','off','MaxHeadSize',0.8)
text(2.2,0,'\bfE','Color','g','FontSize',20); text(2.2,.4,'\bf-','Color','b','FontSize',40)
text(-2.2,.4,'\bf+','Color','r','FontSize',40); x = [0.55 0.52]; y = [0.75 0.54];
annotation('textarrow',x,y,'String','\bfT = p_{e} x E','FontSize',14);
x12 = zeros(steps,2); x13 = zeros(steps,2); y12 = zeros(steps,2); y13 = zeros(steps,2); theta1 = zeros(steps,1);
MM(steps) = struct('cdata',[],'colormap',[]);
for k = 1:steps; theta1(k) = 2*pi*(k-1)/steps; x12(k,1) = 0; x13(k,1) = 0; y12(k,1) = 0; y13(k,1) = 0;
x12(k,2) = r12*cos(Theta(k)); x13(k,2) = -r12*cos(Theta(k)); y12(k,2) = r12*sin(Theta(k)); y13(k,2) = -r12*sin(Theta(k));
set(hPlot,'XData',x12(k,:),'YData',y12(k,:)); set(h3Plot,'XData',x13(k,:),'YData',y13(k,:));
hr=plot(x12(k,2),y12(k,2),'o','MarkerSize',15,'MarkerEdgeColor','k','MarkerFaceColor','r');
hb=plot(-x12(k,2),-y12(k,2),'o','MarkerSize',15,'MarkerEdgeColor','k','MarkerFaceColor','b');
un=x12(k,2)/sqrt((x12(k,2)+0.25)^2+y12(k,2)^2); wn=y12(k,2)/sqrt((x12(k,2)+0.25)^2+y12(k,2)^2);
hq2=quiver3(0,0,0,un,wn,0,'m'); set(hq2,'LineWidth',3,'MaxHeadSize',0.8)
hq3=quiver3(x12(k,2),y12(k,2),0,0.9,0,0,'r'); set(hq3,'LineWidth',3,'MaxHeadSize',0.8)
hq4=quiver3(-x12(k,2),-y12(k,2),0,-0.9,0,0,'b'); set(hq4,'LineWidth',3,'MaxHeadSize',0.8)
hq=quiver3(0,0,0,0,0,-Th*sin(Theta(k)),'k'); set(hq,'LineWidth',3,'MaxHeadSize',0.8)
MM(k) = getframe; if k~=steps; set(hq,'visible','off'); set(hr,'visible','off'); set(hb,'visible','off');
set(hq2,'visible','off'); set(hq3,'visible','off'); set(hq4,'visible','off'); end; view(-73,34); end
```

**1.** Suppose the dipole moment is aligned with E-field vector. Is this equilibrium stable or not? What does it mean that some object at equilibrium? For what angles $\theta$ is the dipole in equilibrium? Briefly explain your answers.

**2.** Explain why the Lorentz's forces applied to the top and bottom charges are directed in the opposite direction?

**3.** Will the shown above dipole rotate clockwise or counterclockwise when released at $t = 0$? When external E-field is on, will the torque on the dipole due to this field tend to align or misalign the electric moment with the field.

**4.** Explain the fact that the dipole released from rest at $t = 0$ with its moment not aligned with E-field, will oscillate back and forth about E-field direction. Check the animation images and the graph in Figure 1.

**5.** Can you predict where the dipole will momentarily stop by looking at the Figure 1 graph and animation? What is the dipole potential and kinetic energy at these moments?

**6.** What is the total net force on the dipole and the direction of torque vector $\boldsymbol{T}$ at $t = 0$? *Hint.* Apply the right-hand-rule.

**7.** What is the unit dimension of torque in SI? Explain spin-electric field energy exchange supporting dipole oscillation.

**8.** Justify the fact that the torque vector $\boldsymbol{T}$ is directed along the axis of the dipole spinning and the cross product of the moment $\boldsymbol{p}_e$ and vector $\boldsymbol{E}$. *Hint.* Use the analogy with the twisting force applied to the wrench and the direction of nut movement along the bolt (see Figure 2.1.1a in Chapter 2).

**9.** Demonstrate that the torque force rotates the dipole in the equilibrium position of minimal potential energy checking the time moment when $\boldsymbol{T} = 0$ in Figure 2.

**10.** Run the case $\theta_0 = 91°$ and dissipation factor = 0. What happened to the dipole oscillation? Why is it not harmonic anymore? Check the case $\theta_0 = 170°$ to be sure and look carefully how the dipole rotates and the torque vector varies.
**11.** Pay attention to the oscillation frequency printed on the top of Figure 1. Why is it so high? Does this frequency increase or decrease as the strength of E-field grows? Explain your answer.
**12.** Eventually, the dipole and E-fields around it rotates jointly being inseparable. Does it mean that the net E-fields is not static any longer and become time dependable? Check Table 1.7 in Chapter 1 and explain the consequences of this effect[2]. *Hint.* Come back to Section 1.6.15 in Chapter 1 and expression (1.62). Should we expect that part of the oscillation energy is lost through the EM radiation?
**13.** Run the Matlab script assuming the dissipation factor = 0.2 and explain the results. Does the aligned dipole produce its own E-field that reduces or increase the external E-field? What happens with multiple dipoles in dielectrics as soon as the external static E-field is applied (Check Figure 2.2.4 in Chapter 2)?
**14.** Following Section 2.2.2 in Chapter 2 and Matlab simulation data explain the phenomenon of polarization. What is the nature of damping effect?
**15.** Calculate the polarization vector magnitude of the material having 300 dipoles per unit volume in volume of 10 units. What is unite dimension of this vector?
**16.** Explain how the displacement vector ***D*** holds unchanged and independent of material dielectric constant. When an object becomes polarized, does it acquire a charge and becomes a charged object?
**17.** Explain the differences between isotropic and anisotropic dielectric materials.

## 2.2 **Rotation the Loop Carrying Steady Current in Uniform B-field**.

Animation https://www.youtube.com/watch?v=aMH7pdn-qr4

[2] David J. Griffiths, , Introduction to Electrodynamics, 3rd Edition, 2007, Pearson Education , Chapter 11, Post 4 and http://physicspages.com/pdf/Griffiths%20EM/Griffiths%20Problems%2011.04.pdf