

To boost the flavor of electromagnetics, we accompanied many problems with short Matlab code acting as a *calculator and result illustrator*. We completely ignored the description of algorithms behind the codes because they are often quite complicated and required ample, very sophisticated as well abstract mathematical specifics poorly helping to understand the physical picture. Such approach let us shift the focus from the often emasculated and practically fruitless problems primarily based on math transformations to more complicated but close to practical tasks. We hope that it helps to visualize typically invisible EM fields and stimulate in-depth analysis and following discussion of the fundamental principles. Projecting images onto a big screen, the lectures may organize whole class conversation making the audience to be significantly engaged.

We cheer our readers to look through Matlab scripts to discover and sophisticate the algorithm embedded in them. We encourage to use the student edition of MATLAB and CST STUDIO SUITE® to get enhanced problem understanding. Sorry for compact transcription of scripts that is dictated by the blog environment.

A better readable version of scripts will be sent to you upon request left on the Contact page.

Attention. Regrettably, copy and paste into Matlab Command Window saves appropriate Matlab format just in Chrome Web Browser. You have to restart <https://emfieldbook.com/> in <https://www.google.com/> if your browser is different.

Section 1.3. Dimensional Analysis and Unit Law

1.1D. Use Table 1.5 to check which of the following equations is dimensionally valid and describes the Lorentz's force equation correctly

- a) $\mathbf{F}_{em} = Q_e(\mathbf{E} + \mathbf{v} \times \mathbf{B})$ b) $\mathbf{F}_{em} = Q_e(\mathbf{D} + \mathbf{v} \times \mathbf{H})$
 c) $\mathbf{F}_{em} = Q_m(\mathbf{E} + \mathbf{v} \times \mathbf{B})$ d) $\mathbf{F}_{em} = Q_e\mathbf{E} + Q_m\mathbf{v} \times \mathbf{B}$

1.2D. Use Table 1.5 to check which of the following equations is dimensionally valid and describes the 1st Maxwell's equation in the differential form correctly

- a) $-\nabla \times \mathbf{E} = \frac{\partial \mathbf{H}}{\partial t} + \mathbf{j}_{mV}$ c) $-\nabla \times \mathbf{E} = \frac{\partial \mathbf{B}}{\partial t} + \mathbf{j}_{eV}$
 b) $-\nabla \times \mathbf{E} = \frac{\partial \mathbf{B}}{\partial t} + \mathbf{j}_{mV}$ d) $-\nabla \times \mathbf{D} = \frac{\partial \mathbf{H}}{\partial t} + \mathbf{j}_{mV}$

1.3D. Use Table 1.5 to check which of the following equations is dimensionally valid and describes the 2nd Maxwell's equation in the differential form correctly

- a) $\nabla \times \mathbf{H} = \frac{\partial \mathbf{E}}{\partial t} + \mathbf{j}_{eV}$ c) $\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{j}_{mV}$
 b) $\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{j}_{eV}$ d) $\nabla \times \mathbf{B} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{j}_{eV}$

1.4D. Use Table 1.5 to check which of the following equations is dimensionally valid and describes the *electric* charge conservation law correctly

- a) $\oint_A \mathbf{j}_{mV} \circ d\mathbf{A} + \frac{dQ_e(t)}{dt} = 0$ c) $\oint_A \mathbf{j}_{eV} \circ d\mathbf{A} + \frac{dQ_m(t)}{dt} = 0$
 b) $\oint_L \mathbf{j}_{eV} \circ d\mathbf{A} + \frac{dQ_e(t)}{dt} = 0$ d) $\oint_A \mathbf{j}_{eV} \circ d\mathbf{A} + \frac{dQ_e(t)}{dt} = 0$

1.5D. The vector \mathbf{H} of *static* magnetic field is measured in base units [A/m] along with some closed path L . Use Table 1.5 to check which of the following equations is dimensionally valid and describes the Ampere's law correctly

- a) $\oint_L \mathbf{H} \circ d\mathbf{L} = I_m$ c) $\oint_L \mathbf{H} \circ d\mathbf{L} = Q_e$

$$b) \oint_L \mathbf{H} \circ d\mathbf{L} = I_e$$

$$d) \oint_L \mathbf{H} \circ d\mathbf{L} = Q_m$$

1.6D. The vector \mathbf{D} of electric displacement field is measured in base units [As/m²] along some closed surface A . Use Table 1.5 to check which of the following equations is dimensionally valid and describes the 3rd Maxwell's equation (Gauss's law) correctly

$$a) \oiint_A \mathbf{D} \circ d\mathbf{A} = I_e$$

$$c) \oiint_A \mathbf{D} \circ d\mathbf{A} = I_m$$

$$b) \oiint_A \mathbf{D} \circ d\mathbf{A} = Q_e$$

$$d) \oiint_A \mathbf{D} \circ d\mathbf{A} = Q_m$$

1.7D. The vector \mathbf{B} of the magnetic inductance field is measured in base units [As/m²] on some closed surface A . Use Table 1.5 to examine which of the following equations is dimensionally valid and describes the 3rd Maxwell's equation (Gauss's law) correctly

$$a) \oiint_A \mathbf{D} \circ d\mathbf{A} = I_e$$

$$c) \oint_L \mathbf{D} \circ d\mathbf{L} = I_e$$

$$b) \oiint_A \mathbf{D} \circ d\mathbf{A} = Q_e$$

$$d) \oiint_A \mathbf{D} \circ d\mathbf{A} = Q_m$$

1.8D. You found reading several publications that power P_Σ emitted by the infinitesimal current element is defined as

$$a) P_\Sigma = \frac{8\pi}{3} \left(\frac{I_e^{exc} \Delta l}{2\sqrt{2}\lambda} \right)^2 / Z_0$$

$$c) P_\Sigma = \frac{8\pi}{3} \left(\frac{I_e^{exc} \Delta l}{2\sqrt{2}\lambda} \right)^2 Z_0$$

$$b) P_\Sigma = \frac{8\pi}{3} \left(\frac{I_e^{exc} \Delta l}{2\sqrt{2}\lambda} \right) Z_0^2$$

$$d) P_\Sigma = \frac{8\pi}{3} \left(\frac{I_e^{exc} \Delta l}{2\sqrt{2}\lambda} \right)^2 Z_0^2$$

Which expression may be correct if Δl is the physical length of the element, I_e^{exc} is the electric current, λ is a wavelength in [m], and Z_0 is the impedance of free space in [Ohms] (check Section 4.2 in Chapter 4 for details)?

1.9D. An inductance L [H] is attached in parallel or series to a capacitance C [F]. Using data from Table 1.5, find such a combination of these values that correctly defines the angular resonance frequency ω [1/s] of such a parallel resonance circuit.

1.10D. The distributed parameters of some transmission line are \mathcal{L}' [H/m] and C' [F/m] (see Sections 6.1 and 6.5 of Chapter 6 for details). Using data from Table 1.5, find such combination of these parameters that correctly defines the characteristic line impedance Z_c [Ohms].

1.11D. Using the first Maxwell's equation in Table 1.7 of Chapter 1 and knowing that \mathbf{E} [V/m] find the dimension unit of magnetic current I_m . Are you surprised? Read Section 1.4.4 and explain the result. Check Law of electromagnetic induction (1.42) for better understanding.

APPENDIX. Vector Differential Operators in Cartesian, Cylindrical and Spherical Coordinate System.

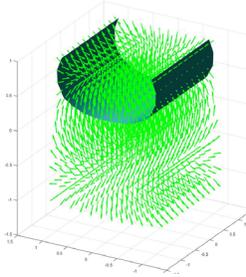
The following Matlab scripts return analytical expressions of gradient of scalar function as well as divergence and curl of vector-function. Wide variety of mathematical functions and their combinations including special functions like Bessel, Legendre, Chebyshev, and many others¹ are accepted. The only requirement is to use the sentence symbolic structure that is recognizable in Matlab Symbolic Toolbox. For example, `besseli(1, r)*`

¹ Look through Matlab documentation or the list in Symbolic Math Tool – Functions at

https://www.mathworks.com/help/symbolic/referencelist.html?searchHighlight=symbolic%20legendre&tid=doc_srchtmltype=function

legendreP (1, cos (theta))*sin (phi) means the modified Bessel function of the first kind times Legendre polynomial times cosine function. These scripts make you very forceful allowing to get and check practically any equation in this and many other book of electrodynamics, antennas, microwave etc. For example, you can apply analytically and get without errors (!) the vector operator curl (curl \mathbf{F}) in spherical coordinate system. Such an act of bravery might be performed by a quite narrow number of humans. Please just look at expression (A.46) on page 473 and correct its left side as $\text{grad}(\text{div } \mathbf{F}) - \nabla^2 \mathbf{F}$. Sorry for the typo.

1A. **Gradient of Scalar Function.** The local function in the Matlab script below is taken from https://www.mathworks.com/matlabcentral/fileexchange/66482-gradient_sym-v-x-coordinate_system. **Warning!** Copy and paste Matlab script below into a newly created file in **Editor Window**, then save it as GradF.m, for example. The presence



of local functions in the script requires Matlab R2016b or later. Push Run button  in Editor Window and proceed with simulation following the instructions on the screen. By definition, the numerical simulation of gradient takes place inside a cube's surface $\max\{|x|,|y|,|z|\}=1$. You can adjust Matlab script if you wish to change it. *Be careful with Matlab syntax entering math expressions!* Choose the Cartesian coordinate system first and run the example depicted on the screen. The force lines of gradient vector $\text{grad } F$ (shown as green cones) cross the isosurfaces (dark green) where $F = \text{const}$. For the sake of better presentation, this constant was chosen 0.5 while $\text{abs}(F) \leq 1$. You can adjust Matlab script if you wish to change it. Pay attention and then explain the mutual orientation of the vector fields and isosurfaces.

```
close all; clc; clear;
n=input('Enter 1 for Cartesian, 2 for Cylindrical, and 3 for Spherical: ');
disp('Follow Matlab rules for math expressions. ');
N=16; [Xq,Yq,Zq]=meshgrid(linspace(-1,1,N)); syms x y z r theta phi
switch n
case 1
    F=input(['Enter scalar fuction of x, y, z (lowercase letters only!) as [Fx, Fy, Fz].!...'
        '\nFor example, cos(pi*y/3)*sin(2*pi*z/3) ']);
    C=gradient_Sym(F,[x,y,z],'Cartesian'); C0=formula(C);
    C1=double(subs(C0(1),[x,y,z],[Xq,Yq,Zq])); C2=double(subs(C0(2),[x,y,z],[Xq,Yq,Zq]));
    C3=double(subs(C0(3),[x,y,z],[Xq,Yq,Zq])); F1=double(subs(F,[x,y,z],[Xq,Yq,Zq]));
case 2
    F=input(['\nEnter scalar fuction of r, phi, z (lowercase letters only!) as [Fr, Fphi, Fz].!...'
        '\nFor example, r*sin(phi) ']);
    R=sqrt(Xq.^2+Yq.^2+Zq.^2); Phi=atan2(Yq,Xq);
    C=gradient_Sym(F,[r,phi,z],'Cylindrical'); C0=formula(C);
    C1=double(subs(C0(1),[r,phi,z],[R,Phi,Zq])); C2=double(subs(C0(2),[r,phi,z],[R,Phi,Zq]));
    C3=double(subs(C0(3),[r,phi,z],[R,Phi,Zq])); F1=double(subs(F,[r,phi,z],[R,Phi,Zq]));
case 3
    F=input(['\nEnter scalar fuction of r, theta, phi (lowercase letters only!) as [Fr, Ftheta, Fphi].!...'
        '\nFor example, r*cos(theta)*sin(phi) ']);
    R=sqrt(Xq.^2+Yq.^2+Zq.^2); Theta=acos(Zq./R); Phi=atan2(Yq,Xq);
    C=gradient_Sym(F,[r,theta,phi],'Spherical'); C0=formula(C);
    C1=double(subs(C0(1),[r,theta,phi],[R,Theta,Phi])); C2=double(subs(C0(2),[r,theta,phi],[R,Theta,Phi]));
    C3=double(subs(C0(3),[r,theta,phi],[R,Theta,Phi])); F1=double(subs(F,[r,theta,phi],[R,Theta,Phi]));
end; fprintf(2, 'The scalar function is:\n'); F
fprintf(2, 'The gradient vector is:\n'); Grad_F=C'
F1=F1/max(abs(F1(:))); figure('units','normalized','outerposition',[0 0 1 1]); grid on, hold on; axis tight
h1=isosurface(Xq,Yq,Zq,F1,0.5,F1); p1=patch(h1); set(p1,'FaceColor','blue','EdgeColor','none','FaceAlpha',0.8);
c=coneplot(Xq,Yq,Zq,C1,C2,C3,Xq,Yq,Zq,0.5); set(c,'FaceColor','g','EdgeColor','g','FaceAlpha',0.5);
camproj perspective; view(-77,34); box on; light; h = camlight('left');
for i = 1:36; camorbit(10,0); camlight(h,'left'); pause(1); end
function gradientSym = gradient_Sym(V,X,coordinate_system); switch coordinate_system
case {'Cartesian'}
    gradientSym=gradient(V,X);
case {'Cylindrical'}
    gradientSym=[diff(V,X(1)),1/X(1)*diff(V,X(2)),diff(V,X(3))];
case {'Spherical'}
    gradientSym=[diff(V,X(1)),1/X(1)*diff(V,X(2)),1/(X(1)*sin(X(2)))*diff(V,X(3))]; end; end
```

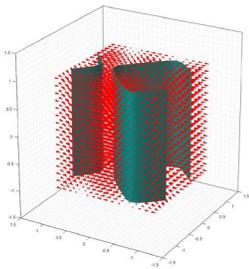
Note that the script not only calculates and plots the vector grad (F) but depicts one of the isosurfaces, i.e. the surface where $F = \text{const}$. For the sake of better presentation, this constant was chosen 0.5 while the function F is normalized to $\text{abs}(F) \leq 1$. You can correct the script if you wish. Pay attention and then explain the mutual orientation of the vector fields and isosurfaces.

Now several questions to check the understanding. Please refresh the material in Appendix, pages 469 - 476.

1. Why the scalar function gradient is a vector and what does it means?
2. What does the gradient of function indicate?
3. What does it mean if the function gradient is zero at some point?
4. How the gradient link to the function slope?
5. Suppose the orientation of gradient vector is positive at some point. What does it tell us about the function?
6. What is the orientation of gradient vector over the isosurface $F = \text{const}$.? Check your answer looking at the plot.
7. Assuming that the function $F = U$ is the potential U of E-field (see (1.21) in Chapter 1), apply *curl* operator from 3A to calculated $\mathbf{E} = -\text{Grad}_F$ and check that $\text{curl}(\text{grad } U) \equiv 0$ in any of coordinate systems. What does it tell us about E-fields, their orientation in space, and ability to move charges (check (A.50) – (A.54))?
8. Assuming again that the function $F = U$ is the potential U of E-fields, apply the *divergence* operator from 2A to calculated $\text{div } \mathbf{E} = \text{div}(\text{grad } U) = \nabla^2 U$ and thereby find the distribution of the E-fields' sources, i.e. the volume density of electric charges (check (A.21) on page 470).

2A. **Divergence of Vector Function.** The local function in the Matlab script below is taken from

https://www.mathworks.com/matlabcentral/fileexchange/66259-divergence_sym-v-x-coordinate_system. **Warning!** Copy and paste Matlab script below into a newly created file in **Editor Window**, then save it as DivF.m, for example. The presence

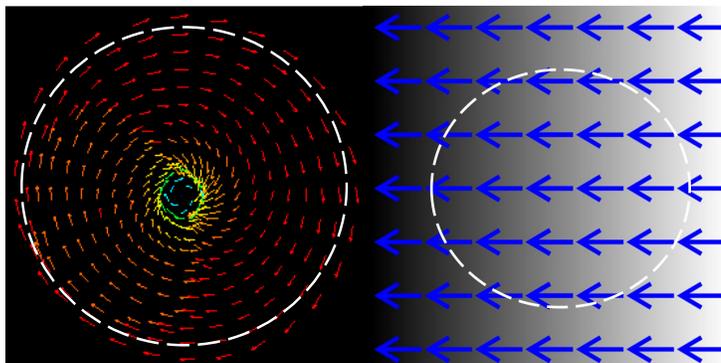


of local functions in the script requires Matlab R2016b or later. Push Run button  in Editor Window and proceed with simulation following the instructions on the screen. By definition, the numerical simulation of divergence takes place inside a cube's surface $\max\{|x|,|y|,|z|\}=1$. *Be careful with Matlab syntax entering math expressions!* Choose the Cartesian coordinate system first and run the example

depicted on the screen. The force lines of vector function \mathbf{F} (shown as red cones) cross the cylindrical isosurfaces (dark green) where $\text{div } \mathbf{F} = \text{const}$. For the sake of better presentation, this constant was chosen 0.05 while $\text{abs}(\text{div } \mathbf{F}) \leq 1$. You can adjust Matlab script if you wish to change it. Pay attention and then explain the mutual orientation of the vector fields and isosurfaces.

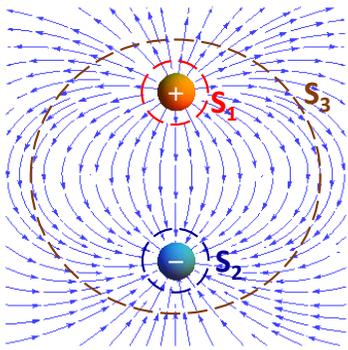
Now several questions to check the understanding. Please refresh the material in Appendix, pages 469-470 and Gauss' Theorem on page 477.

1. Is the divergence of vector function is a vector or scalar? Using the analogy of water stream interpret the concept of “outflowing-ness” and “inflowing-ness.”



2. We measured the net flux of 2D E-forces lines (red and blue arrows on the left) crossing the surface (white dot line) of cylinder. Which of the images corresponds to zero flux?

3. What does it mean that the divergent of E-field is positive in all points inside some volume? Can we expect the presence of sources or sinks in this volume? What does it



mean that the divergent of E-field is negative in all points inside some volume? Can we expect the presence of sources or sinks in this volume?

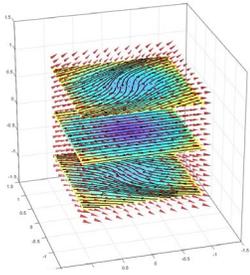
4. Give more examples of E-fields with zero, positive and negative divergence and flux illustrating your answer with E-force lines.

5. The plot on the left depicts 2D E-force lines (blue arrows) starting from the positive charge (red ball) and ending at the negative charge (blue ball). What could you tell about the sing of divergence of E-field vector and its flux through the surface of cylinders S_1 , S_2 , and S_3 ?

```
close all; clc; clear; n=input('Enter 1 for Cartesian, 2 for Cylindrical, and 3 for Spherical: ');
disp('Follow Matlab rules for math expressions. Square brackets [] are required.')
N=16; [Xq,Yq,Zq]=meshgrid(linspace(-1,1,N)); syms x y z theta phi
switch n
case 1
    F=input(['Enter Vector-fuction of x, y, z (lowercase letters only!) as [Fx, Fy, Fz].'...
        '\nFor example, [sin(pi*x), cos(pi*y), 0] ']);
    C=divergence_sym(F,[x,y,z],'Cartesian');
    C1=double(subs(C,[x,y,z],[Xq,Yq,Zq])); F1=double(subs(F(1),[x,y,z],[Xq,Yq,Zq]));
    F2=double(subs(F(2),[x,y,z],[Xq,Yq,Zq])); F3=double(subs(F(3),[x,y,z],[Xq,Yq,Zq]));
case 2
    F=input(['Enter vector-fuction of r, phi, z (lowercase letters only!) as [Fr, Fphi, Fz].'...
        '\nFor example, 2*r^2*[cos(2*phi) -sin(2*phi) 1] ']);
    R=sqrt(Xq.^2+Yq.^2+Zq.^2); Phi=atan2(Yq,Xq); Z=Zq;
    C=divergence_sym(F,[r,phi,z],'Cylindrical'); F0=formula(F);
    C1=double(subs(C,[r,phi,z],[R,Phi,Z])); F1=double(subs(F0(1),[r,phi,z],[R,Phi,Z]));
    F2=double(subs(F0(2),[r,phi,z],[R,Phi,Z])); F3=double(subs(F0(3),[r,phi,z],[R,Phi,Z]));
case 3
    F=input(['Enter vector-fuction of r, theta, phi (lowercase letters only!) as [Fr, Ftheta, Fphi].'...
        '\nFor example, [cos(theta)/r sin(theta)/r 0] ']);
    R=sqrt(Xq.^2+Yq.^2+Zq.^2); Theta=atan2(sqrt(Xq.^2+Yq.^2),Zq); Phi=atan2(Yq,Xq); Z1=Zq;
    C=divergence_sym(F,[r,theta,phi],'Spherical'); F0=formula(F);
    C1=double(subs(C,[r,theta,phi],[R,Theta,Phi])); F1=double(subs(F0(1),[r,theta,phi],[R,Theta,Phi]));
    F2=double(subs(F0(2),[r,theta,phi],[R,Theta,Phi])); F3=double(subs(F0(3),[r,theta,phi],[R,Theta,Phi]));
end
fprintf(2, 'The vector-function is:\n'); F
fprintf(2, 'The divergence is:\n'); Div_F=C
C1=C1/max(abs(C1(:)));
figure('units','normalized','outerposition',[0 0 1 1]); grid minor, hold on; axis square
c=coneplot(Xq,Yq,Zq,F1,F2,F3,Xq,Yq,Zq,0.5);
set(c,'FaceColor','r','EdgeColor','r','FaceAlpha',0.5);
isosurface(Xq,Yq,Zq,C1,0.05)
camproj perspective; view(-60,22); box on; light; h = camlight('left');
for i = 1:36; camorbit(10,0); camlight(h,'left'); pause(1); end
function Div = divergence_sym(V,X,coordinate_system)
switch coordinate_system
case {'Cartesian'}
    Div=divergence(V,X);
case {'Cylindrical'}
    Div=1/X(1)*diff(X(1)*V(1),X(1))+1/X(1)*diff(V(2),X(2))+diff(V(3),X(3));
case {'Spherical'}
    Div=1/X(1)^2*diff(X(1)^2*V(1),X(1))+1/(X(1)*sin(X(2)))*diff(sin(X(2))*V(2),X(2))+1/(X(1)*sin(X(2)))*diff(V(3),X(3));
end; end
```

3A. **Curl of Vector Function.** The local function in the Matlab script below is taken from

https://www.mathworks.com/matlabcentral/fileexchange/66265-curl_sym-v-x-coordinate_system. **Warning!** Copy and paste Matlab script below into a newly created file in **Editor Window**, then save it as CurlF.m, for example. The presence

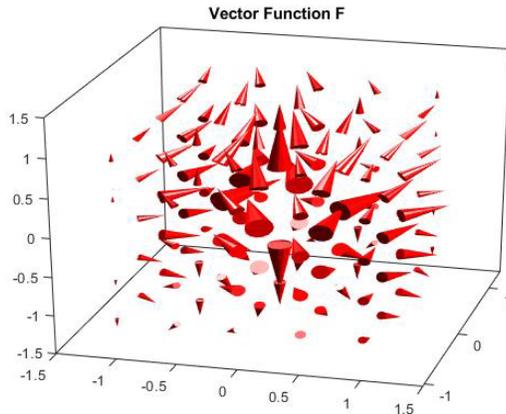


of local functions in the script requires Matlab R2016b or later. Push Run button  in Editor Window and proceed with simulation following the instructions on the screen. By definition, the numerical simulation of curl takes place inside a cube's surface $\max\{|x|,|y|,|z|\}=1$. You can adjust Matlab script if you wish to change it. *Be careful with Matlab syntax entering math expressions!* Choose the Cartesian coordinate system first and run the example depicted on the screen. The normalize profile of $\text{abs}(\mathbf{F}) < 1$ is shown in three cross section $z = -0.8, 0, 0.8$. The force lines of vector function \mathbf{F} are depicted in red while the force lines of vector $\text{curl}\mathbf{F}$ are black.

```
close all; clc; clear;
n=input('Enter 1 for Cartesian, 2 for Cylindrical, and 3 for Spherical: ');
disp('Follow Matlab rules for math expressions. Square brackets [] are required.')
N=12; [Xq,Yq,Zq]=meshgrid(linspace(-1,1,N)); syms x y z r theta phi
switch n
case 1
F=input(['Enter Vector-fuction of x, y, z (lowercase letters only!) as [Fx, Fy, Fz].'...
'\nFor example, [0,0,cos(pi*y/2)*cos(pi*z/2) ]]); C=curl_sym(F,[x,y,z],'Cartesian');
C1=double(subs(C(1),[x,y,z],[Xq,Yq,Zq])); C2=double(subs(C(2),[x,y,z],[Xq,Yq,Zq]));
C3=double(subs(C(3),[x,y,z],[Xq,Yq,Zq])); F1=double(subs(F(1),[x,y,z],[Xq,Yq,Zq]));
F2=double(subs(F(2),[x,y,z],[Xq,Yq,Zq])); F3=double(subs(F(3),[x,y,z],[Xq,Yq,Zq]));
case 2
F=input(['\nEnter vector-fuction of r, phi, z (lowercase letters only!) as [Fr, Fphi, Fz].'...
'\nFor example, [-r*sin(phi),r*cos(phi),0] ');
R=sqrt(Xq.^2+Yq.^2+Zq.^2); Phi=atan2(Yq,Xq); Z=Zq; C=curl_sym(F,[r,phi,z],'Cylindrical');
C1=double(subs(C(1),[r,phi,z],[R,Phi,Z])); C2=double(subs(C(2),[r,phi,z],[R,Phi,Z]));
C3=double(subs(C(3),[r,phi,z],[R,Phi,Z])); F1=double(subs(F(1),[r,phi,z],[R,Phi,Z]));
F2=double(subs(F(2),[r,phi,z],[R,Phi,Z])); F3=double(subs(F(3),[r,phi,z],[R,Phi,Z]));
case 3
F=input(['\nEnter vector-fuction of r, theta, phi (lowercase letters only!) as [Fr, Ftheta, Fphi].'...
'\nFor example, [2*r*cos(theta),-3*r*sin(theta),r*cos(phi) ]]);
R=sqrt(Xq.^2+Yq.^2+Zq.^2); Phi=atan2(Yq,Xq); Theta=atan2(sqrt(Xq.^2+Yq.^2),Zq);
C=curl_sym(F,[r,theta,phi],'Spherical'); C0=formula(C); C1=double(subs(C0(1),[r,theta,phi],[R,Theta,Phi]));
C2=double(subs(C0(2),[r,theta,phi],[R,Theta,Phi])); C3=double(subs(C0(3),[r,theta,phi],[R,Theta,Phi]));
F1=double(subs(F(1),[r,theta,phi],[R,Theta,Phi])); F2=double(subs(F(2),[r,theta,phi],[R,Theta,Phi]));
F3=double(subs(F(3),[r,theta,phi],[R,Theta,Phi])); end
disp('You entered function F'); F'
FF=sqrt(F1.^2+F2.^2+F3.^2); FF=FF/max(abs(FF(:))); Curl_F=C'
figure('units','normalized','outerposition',[0 0 1 1]); grid on, hold on; axis square
if n==2; quiver3(Xq,Yq,Zq,C1,C2,C3,1,'LineWidth',2,'MaxHeadSize',1,'color','g'); end
hs1=slice(Xq,Yq,Zq,FF,[],[],[-0.8,0,0.8]); set(hs1,'FaceColor','interp','EdgeColor','none');
if n~=2; hs2=streamslice(Xq,Yq,Zq,C1,C2,C3,[],[],[-0.8,0,0.8],1); set(hs2,'Color','k','LineWidth',2); end
c=coneplot(Xq,Yq,Zq,F1,F2,F3,Xq,Yq,Zq,0.5); set(c,'EdgeColor','none','FaceColor','r');
camproj 'perspective'; view(-106,26); box on; light; h = camlight('left');
for i = 1:36; camorbit(10,0); camlight(h,'left'); pause(1); end
function CurlSym = curl_sym(V,X,coordinate_system)
switch coordinate_system
case {'Cartesian'}
CurlSym=curl(V,X);
case {'Cylindrical'}
CurlSym=[1/X(1)*diff(V(3),X(2))-diff(V(2),X(3)),...
diff(V(1),X(3))-diff(V(3),X(1)),...
1/X(1)*diff(X(1)*V(2),X(1))-1/X(1)*diff(V(1),X(2))];
case {'Spherical'}
CurlSym=[1/(X(1)*sin(X(2)))*(diff(V(3)*sin(X(2)),X(2))-diff(V(2),X(3))),...
1/X(1)*(1/sin(X(2))*diff(V(1),X(3))-diff(X(1)*V(3),X(1))),...
1/X(1)*(diff(X(1)*V(2),X(1))-diff(V(1),X(2)))]];
end; end
```

Now several questions to check the understanding. Please refresh the material in Appendix, pages 471, 475 - 477.

4A. Electric Potential of Conservative and Non – Conservative E-Fields. Please refresh the material of



Sections 1.6.1 – 1.6.3 in Chapter 1, Section 2.2.8 in Chapter 2, and the comments on pages 475 – 476 in Appendix. **Warning!** Copy and paste Matlab script below into a newly created file in **Editor Window**, then save it as Potential.m, for example. The presence of local functions in the script requires Matlab R2016b or later. Push Run button  in Editor Window and proceed with simulation following the instructions on the screen. *Be careful with Matlab syntax entering math expressions!* The script animates the effect of the end point 2 (check expression (1.21) of Chapter 1), and the path taken between point 1 and 2 on the work done by E-field down the path. For the sake of simplicity, starting point 1 for both paths

is fixed while point 2 (red or blue) moves along the designated curve, helical segment (red) or vertical line (blue), respectively. The end point 2 of both curves is shared.

```
close all; clc; clear; syms x y z real; R0=sqrt(x^2+y^2+z^2);
Rp=sqrt((x+0.25)^2+(y+0.25)^2+(z+0.25)^2); Rm=sqrt((x-0.25)^2+(y-0.25)^2+(z-0.25)^2);
fprintf(2,['Input below Vector-fuction E(x,y,z) (lowercase letters for x,y,z) as [Ex, Ey, Ez]:'...
'\nYou can type R0=sqrt(x^2+y^2+z^2); Rp=sqrt((x+0.25)^2+(y+0.25)^2+(z+0.25)^2);'...
'\nRm=sqrt((x-0.25)^2+(y-0.25)^2+(z-0.25)^2); \n as variables. For example,' ...
'\n [x-0.25,y-0.25,z-0.25]/Rm^3-[x+0.25,y+0.25,z+0.25]/Rp^3' ...
'\n or [x^2-y^2,abs(y-x),z]/R0^2 \n\n']); E=input('Enter Vector-function here E = ');
figure(1); vectorfield3(E,-1:.5:1,-1:.5:1,-1:.5:1); camproj perspective; box on; light; camlight('left');
title('\bfE-force Line Durection and Magnitude'); view(15,25);
Curl=curl(E,[x y z]); CurlMag=simplify((Curl(1)^2+Curl(2)^2+Curl(3)^2));
cond = subs(CurlMag,{x,y,z},{1,1,1}); if cond==0; fprintf(2,'E-Field is Concervative\n'); else
fprintf(2,'E-Field is Non-Concervative\n'); end; H=[cos(t),sin(t),t]; H_1=[1,0,t];
Fpar=subs(E,[x,y,z],H); Fpar_1=subs(E,[x,y,z],H_1);
F_dl=simplify(dot(Fpar,diff(H,t))); F_dl_1=simplify(dot(Fpar_1,diff(H_1,t))); ti=0:pi/20:4*pi;
Curve_1=double(subs(H,t,ti)); Curve_1=double(subs(H_1,t,ti)); L=length(ti);
xi=Curve(1:L); yi=Curve(L+1:2*L); zi=Curve(2*L+1:end);
xi1=Curve_1(1:L); yi1=Curve_1(L+1:2*L); zi1=Curve_1(2*L+1:end);
for i=1:length(ti); integral(i)=quadl(inline(vectorize(F_dl)),1e-3,ti(i));
integral_1(i)=quadl(inline(vectorize(F_dl_1)),1e-3,ti(i)); end
f2=figure; movegui(f2,'west'); hold all;
camproj perspective; box on; light; h = camlight('left');
plot3(xi,yi,zi,'r',xi1,yi1,zi1,'b'); view(3); title('\bfPath along Helix and Line')
text(xi(1),yi(1),zi(1),'\bf Point 1'); text(xi(end),yi(end),zi(end),'\bf Point 2');
for i=1:length(ti); h=scatter3(xi(i),yi(i),zi(i),'filled','r'); h.SizeData = 50;
h1=scatter3(xi1(i),yi1(i),zi1(i),'filled','b'); h1.SizeData = 50; drawnow; view(-60,22); pause(0.1)
end
legend('\bfHelical Curve', '\bfVertical Path', 'Location', 'Best'); hold off;
f3=figure; movegui(f3,'east'); hold all; grid minor
plot(ti,integral,'r',ti,integral_1,'b')
title('\bfRelative Electric Potential/Work along Helix and Line')
text(ti(end),integral(end),'\bf Point 2'); text(ti(end),integral_1(end),'\bf Point 2');
for i=1:length(ti); h=scatter(ti(i),integral(i),'filled','r'); h.SizeData = 50;
h1=scatter(ti(i),integral_1(i),'filled','b'); h1.SizeData = 50;
drawnow; pause(0.1); end
legend('\bfHelical Curve', '\bfVertical Path', 'Location', 'Best'); hold off
function vectorfield3(F,xval,yval,zval); [xg,yg,zg]= meshgrid(xval,yval,zval);
F1f = inline(vectorize(F(1)),'x','y','z'); F2f = inline(vectorize(F(2)),'x','y','z');
F3f = inline(vectorize(F(3)),'x','y','z'); F1g = F1f(xg,yg,zg); F2g = F2f(xg,yg,zg); F3g = F3f(xg,yg,zg);
c=coneplot(xg,yg,zg,F1g,F2g,F3g,xg,yg,zg,1); set(c,'EdgeColor','none','FaceColor','r');
camproj perspective; box on; light; camlight('left'); end
```

Run one of the E-field vector example depicted on the screen. The first example corresponds to E-field of two point charges while the second one was chosen as a fiction non-conservative field. The central Figure 1 illustrates E-force lines of vector function \mathbf{E} you have entered. The cone sizes are proportional to E-field intensity. The appeared on display the red message enlightens the status of this field. Figure 3 illustrates how the potential U_e of E-field varies along the curves. The movement of red and blue markers in Figure 2 and 3 is synchronized.

Sections 1.6.1 - 1.6.9, 1.6.17. Static Electric Fields

1.1E. What is the strength and direction of the static and uniform electric field \mathbf{E} [V/m] (use expression (1.17)) pushing an electron of $1.60217662 \times 10^{-19}$ [C] in the positive direction of z-axis by the force 1×10^{15} [N]? Compare this field strength with the air breakdown strength 3×10^6 [V/m]. Might we watch light spike in air or corona as multiple electrons are accelerated simultaneously? *Hint.* Search Wikipedia.

1.2E. Using data from the problem 1.1E and Newton's second law of motion $\mathbf{F} = m\mathbf{a}$, find the electron acceleration of $m = 9.10938356 \times 10^{-31}$ [kg], its velocity, and traveled distance for a time period of 10^{-8} [s]. Compare this velocity with speed of light $c = 299\,792\,458$ [m/s]. Is it possible for such an accelerating electron to reach the speed of light? If your answer is yes, how long does it take?

1.3E. Using data from the problem 1.1E, find the energy W_e [J] (use the expression (1.19) in Chapter 1) that the electric field transfers into the kinetic energy of accelerating an electron. Calculate the kinetic energy of the electron as $W_k = mv^2/2$ and check that $W_k = W_e$. Recalculate W_e in the electron-volt [eV] assuming that $1 \text{ [eV]} = 1.60 \times 10^{-19}$ [J] and explain the meaning of electron-volt units.

1.4E. Using data from the problem 1.1E - 1.3E find the electric potential U_e [V] (use expression (1.21) in Chapter 1) that accelerated the electron.

1.5E. Copy, paste and execute the following script into Matlab Command Window to check your calculations.

```
clc; close all; clear; e = 1.60217662e-19; m = 9.10938356e-31; c = 299792458; F=1e-13; t=1e-9;
E=F/e;a=F/m;v=a*t;ratio=v/c;dist=a*t^2/2;We=e*E*dist;Wk=m*v^2/2;WeV=We/1.6e-19;Ue=We/e;
X1 = ['|E| = ',num2str(E),' V/m;', ' a = ',num2str(a),' m/s^2;', ' v = ',num2str(v),' m/s;'];disp(X1);
X2 = ['v/c = ',num2str(ratio),';', ' dist = ',num2str(dist),' m;', ' We = ',num2str(We),' J;'];disp(X2);
X3 = ['Wk = ',num2str(Wk),' J;', ' WeV = ',num2str(WeV),' eV;', ' Ue = ',num2str(Ue),' V;'];disp(X3);
```

1.6E. Sketch out the equipotential lines of electric potential U_e and lines of E-force produced by static point charges located inside the cube $|x| \leq 2$ [m], $|y| \leq 2$, $|z| \leq 2$ [m]. For example, **a**) single charge in origin $Q=[1]$, $x=[0]$, $y=[0]$, $z=[0]$, **b**) dipole in XY-plane $Q=[1,-1]$, $x=[0,0]$, $y=[1,-1]$, $z=[0,0]$, and **c**) quadrupole in XY-plane $Q=[1,-1,1,-1]$, $x=[-1,1,1,-1]$, $y=[1,1,-1,-1]$, $z=[0,0,0,0]$. Copy, paste, and execute the following script (see next page) into Matlab Command Window. Compare the plots with your sketches and clarify the differences if you found them.

1. Explain how to estimate the relative strength and direction of E-field (see Section 1.6.3 in Chapter 1) using this plot.

2. Show that the vector of E-field from point charges can be obtained by taking the vector sum of the E-fields of the individual charges.

3. Find the total flux of E-fields flowing through the surface of the sphere surrounding each of the charges only and their different combinations (see Sections 1.6.4 and 1.6.8 in Chapter 1).

4. Ask the instructor for additional questions.

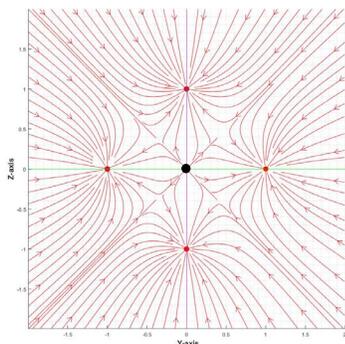
```

clc; close all; clear
fprintf(2,'All charges must be put inside volume -2<=X<=2,-2<=Y<=2, -2<=Z<=2 [m]\n')
fprintf(2,'\n Total number of charges is equal M \n ')
fprintf(2,'\n For example, for dipole in XY-plane M=2 and Vector Q is [1,-0.5]\n')
fprintf(2,'\n Vector X is [0.5 0.5], vector Y is [0.5 -0.2], and vector Z is [0 0] \n')
fprintf(2,'\n Square brackets and Commas are mandatory\n')
fprintf(2,'\n Hint. Before entering data draw all charges and write their coordinate-vectors on paper in required format\n\n')
Q=input('Enter all charges Q in [C] as a vector [Q(1),Q(2),Q(3),...,Q(M)]= ');
xx=input('Enter all X-coordinates in [m] as a row-vector [x(1),x(2),x(3),...,x(M)]= ');
yy=input('Enter all Y-coordinates in [m] as a row-vector [y(1),y(2),y(3),...,y(M)]= ');
zz=input('Enter all Z-coordinates in [m] as a row-vector [z(1),z(2),z(3),...,z(M)]= ');
k=8.987E9; N=150; x=linspace(-1,1,N)*2; y=linspace(-1,1,N)*2; z=linspace(-1,1,N)*2;
[X,Y,Z]=meshgrid(x,y,z); Ue=zeros(size(X)); for ii=1:numel(Q)
hy=sqrt((xx(ii)-X).^2+(yy(ii)-Y).^2+(zz(ii)-Z).^2); Ue=Ue+Q(ii)./hy; end
Ue=k*Ue; V=log(abs(Ue)); [Ex,Ey,Ez]=gradient(Ue,2/N); E=sqrt(Ex.^2+Ey.^2+Ez.^2); LE=log(E);
figure('units','normalized','outerposition',[0 0 1 1]); grid minor; hold on;
h3 = streamslice(X,Y,Z,Ex,Ey,Ez,0,[],[],2);set(h3,'Color','r'); h4 = streamslice(X,Y,Z,Ex,Ey,Ez,[],0,[],2);set(h4,'Color','m')
h5 = streamslice(X,Y,Z,Ex,Ey,Ez,[],[],0,2);set(h5,'Color','g'); for jj=1:numel(Q); if Q(jj)>0;
plot3(xx(jj),yy(jj),zz(jj),'o','MarkerSize',10,'MarkerFaceColor','r'); else
plot3(xx(jj),yy(jj),zz(jj),'o','MarkerSize',10,'MarkerFaceColor','b'); end; end
xlabel('X-axis','FontWeight','bold','FontSize',14); ylabel('Y-axis','FontWeight','bold','FontSize',14);
zlabel('Z-axis','FontWeight','bold','FontSize',14); axis square; view(-74,16);
title('\bfE-force Lines in XY-, XZ- and YZ-plane');
figure('units','normalized','outerposition',[.2 .2 .6 .6]); grid minor; hold on;
colormap(jet(64)); hh=colorbar; ylabel(hh,'\Ln(|Ue(x,y,z)| [V])','FontWeight','Bold','FontSize',15);
h1 = contourslice(X,Y,Z,V,[],0,[],100); h2 = contourslice(-X,Y,Z,V,0,[],[],100);
xlabel('X-axis','FontWeight','bold','FontSize',14); ylabel('Y-axis','FontWeight','bold','FontSize',14);
zlabel('Z-axis','FontWeight','bold','FontSize',14); axis square; view(-69,27);
title('\bfEquipotential Lines of Potential Ue in XZ- and YZ-plane';Logarithmic scale');
vert = [-2 0 -2;2 0 -2;2 0 -2;-2 0 2;-2 0 2;2 0 2;-2 0 2];
fac = [1 2 6 5;1 2 6 5;1 2 6 5;1 2 6 5;1 2 6 5;1 2 6 5];
ph=patch('Vertices',vert,'Faces',fac,'FaceVertexCData',hsv(6),'FaceColor','flat');alpha(ph,0.1)
vert = [0 -2 -2;0 2 -2;0 2 -2;0 -2 2;0 2 2;0 2 2;0 -2 2];
fac = [1 2 6 5;1 2 6 5;1 2 6 5;1 2 6 5;1 2 6 5;1 2 6 5];
ph1=patch('Vertices',vert,'Faces',fac,'FaceVertexCData',hsv(6),'FaceColor','flat');alpha(ph1,0.1)
v1=[0,0,2]; v2=[0,0,-2]; v=[v1;v2]; plot3(v(:,1),v(:,2),v(:,3),'-k','LineWidth',2)
figure('units','normalized','outerposition',[.3 .3 .6 .6]); grid minor; hold on;
colormap(jet(64)); hh=colorbar; ylabel(hh,'\Ln(|E(x,y,z)| [V/m])','FontWeight','Bold','FontSize',15);
contourslice(X,Y,Z,LE,[],[],zz,80); contourslice(-X,Y,Z,LE,0,[],[],80);
xlabel('X-axis','FontWeight','bold','FontSize',14); ylabel('Y-axis','FontWeight','bold','FontSize',14);
zlabel('Z-axis','FontWeight','bold','FontSize',14); axis square; view(-21,19);
title('\bfEquipotential Lines of E-field Logarithm');

```

1.7E. <http://www.falstad.com/vector3de/> in your browser and press Enter. Click “Full-screen version” on the bottom of the opened page. In the “Field selection” on the right top corner choose “dipole” and “Display: Field Lines or Equipotentials.” Go to “Show Z Slice” and compare with the plots you have received solving problem 1.5E. Examine different combinations of charges and material bodies. Explain images. Play and enjoy.

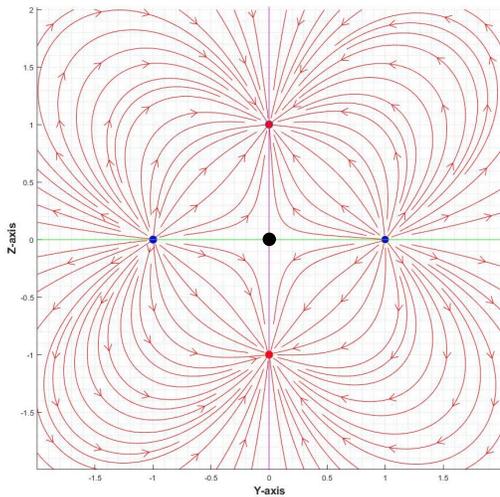
1.8E. The script from the problem 1.6E was run into Matlab Command Window. The figure depicts the obtained image of E-force lines initiated by the quadrupole, i.e. 4 charges are arranged on the corners of a square. The black sport in origin is a motionless field sensor #1 (see Section 1.4.1 in Chapter 1). After analyzing the drawing answer the questions:



1. What is the charges' polarity? Which one is positive and negative?
2. What could you tell about the charges' values?
3. What force might move this sensor and where?
4. How much is the E-field flux through the closed surface that includes all charges?

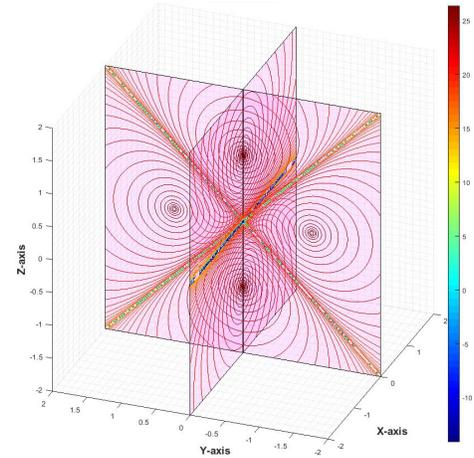
5. How much is the E-field flux through the closed surface that includes the sensor only?

1.9E. The script from the problem 1.6E was rerun into Matlab Command Window. Figure on the left depicts



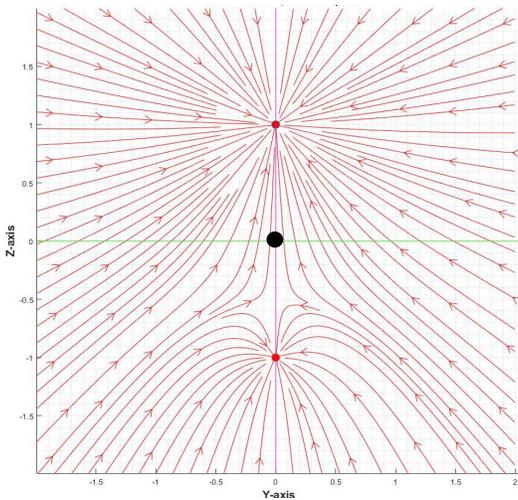
the obtained image of the E-force lines initiated by the quadrupole.

Analyzing the drawing, answer the questions from the problem 1.8E. Then look at the picture on your right that illustrates the equipotential lines of electric potential (really the natural

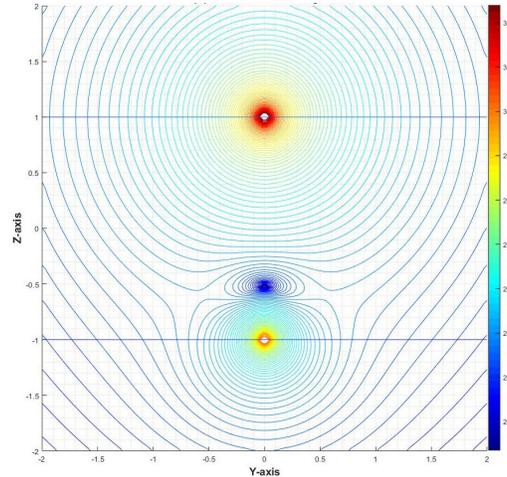


logarithm to get the more revealing image) for the same quadrupole. What is the potential value along the two diagonals in the YZ-plane and along the line $Z = 0$ in the XZ-plane? Suppose the infinitely thin sheets of copper are put along the diagonals in parallel to the y -axis. Does it change the E-field structure and how? (*Hint*. Check boundary conditions for the E-field tangential components in Table 2.2 of Chapter 2). Give detailed explanations and supporting drawing of E-force lines.

1.10E. The script from the problem 1.6E was rerun into Matlab Command Window. Figure on the left depicts



the obtained image of the E-force lines initiated by the two charges. Analyzing the drawing, answer the questions from the problem 1.8E. Then look at the picture on your right that illustrates the equipotential lines of E-fields



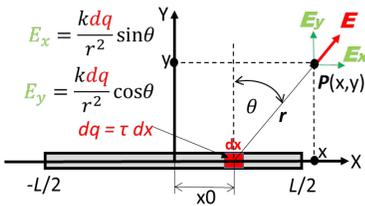
(really the natural logarithm to get the more revealing image). What could you tell about the charges' value? How to explain the deep drop in E-field intensity (blue spot) and its location shifted to one of the charges?

1.10E. Which one of the shown below images of E-force lines reflects correctly a physically sound E-field structure? (*Hint*. Read Section 1.6.3 and check the E-field definition (1.18) in Chapter 1)? Give reasons supporting your answer.



a) b) c)

1.11E. **E-near-fields of the electrically short non-conducting rod.** Suppose a charge Q spreads uniformly along the x -axis with linear density $\tau = Q/L$ where L is the total length of the charged line². E-field due to the total charge Q can be calculated by superposing (really integrating) the E-field of infinitesimal charge dq using the equation (1.24) in Chapter 1. The trigonometric functions are equal $\sin \theta = (x - x_0)/r$ and $\cos \theta = y/r$ where $r^2 = (x - x_0)^2 + y^2$. Allowing x_0 vary from $-L/2$ to $L/2$ write down the integral for each component of E-field. Sketch out the 2D equipotential lines of E-force produced by the charged line. Then copy, paste, and run the following script into Matlab Command Window and compare the plot with your sketch. Explain differences if you found them.



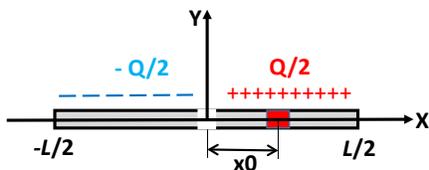
```

clc; close all; clear
k = 8.987E9; display(' Length L of finite line of charge L<=9m')
L=input('Enter the length L of finite line of charge in [m]: ');
tau=input('Enter the charge density of charges in [C/m]: ');
[X, Y] = meshgrid(-2*L:0.020011:2*L); x0=linspace(-L/2,L/2,101);
for jj=1:size(x0,2); r=sqrt((x0(jj)-X).^2+Y.^2); dV(jj,:)=k*tau./r; end
Ue=squeeze(trapz(dV,1)); [Ex,Ey]=gradient(Ue);
F1=figure('units','normalized','outerposition',[0 0 1 1]); grid minor; hold all
hLines = streamslice(X,Y,Ex,Ey,5); set(hLines,'Color','r'); xlabel('-5 \leq X \leq 5','FontWeight','bold');
ylabel('-5 \leq Y \leq 5','FontWeight','bold'); axis tight
plot(x0,0,'ko','MarkerSize',20,'MarkerEdgeColor','k','MarkerFaceColor','k')
annotation(F1,'textarrow',[0.42 0.48],[0.42 0.50],'TextLineWidth',2,'FontSize',20,'FontWeight','Bold',...
'String',{'Charged Line'},'HeadStyle','deltoid','LineWidth',2,'Color','k');
contour(X,Y,Ue,40); hh=colorbar('fontsize',12);
ylabel(hh,'Potential Intensity Ue(X,Y)','FontWeight','Bold','FontSize',15);
xlabel('X-axis [m]','FontSize',15,'FontWeight','bold'); ylabel('Y-axis [m]','FontSize',15,'FontWeight','bold');
axis square; hold off; figure(2); h2=mesh(X,Y,Ue); g2=colorbar('fontsize',12);
xlabel(g2,'Electric Potential Ue [V]','FontWeight','Bold','FontSize',15);
xlabel('x-axis','fontsize',14); ylabel('y-axis','fontsize',14); zlabel('Electric Potential [V]','fontsize',14)
h3=title('Electric Potential Distribution, V(x,y) [V]'); set(h3,'fontsize',14); view(121,24); grid minor

```

1. Examine two individual cases of the short line ($L = 0.1\text{m}$, for example) and relatively long line of $L = 9\text{m}$. Compare plots and explain the differences in E-field structures.
2. What kind of E-field structure could you expect in case of an infinitely long line as $L \rightarrow \infty$?
3. Examine the case of a negative charge.
4. Find the total flux of E- and D-fields flowing through the surface of the sphere surrounding the line of charges.
5. If you like a challenge present in closed form all integrals numerically calculated in the script. Ask the instructor for additional questions.

1.12E **E-near-fields of the electrically short dipole.** Assume that the charge line comprises two separate halves carrying equal but opposite charges $\pm Q/2$. Sketch out the 2D equipotential lines of E-force produced by this charged lines. Then copy, paste, and run the script below into Matlab Command Window. Explain differences if you found them.



² Bear in mind that the charges of the same sign “hate” each other by pushing them away that makes practically impossible to keep such charges in static position at the same area. That is why all the following exercises with static or defined upfront charge distribution are just intended to demonstrate and clear up some physical principles and generally serve as rough approximation of real-world problems.

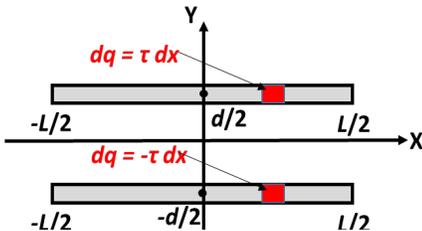
```

clc; close all; clear; k = 8.987E9; a=5;
display(' Length L of finite line of charge L<=9m'); L=input('Enter the length L of finite line of charge in [m]: ');
tau=input('Enter the charge linear density of charges in [C/m]: ');
[X,Y] = meshgrid(-L:0.02:L); x0=linspace(-L/2,L/2,200);
for jj=1:size(x0,2); r(jj,:)=sqrt((x0(jj)-X).^2+Y.^2); x1(jj,:)=X-x0(jj);
    if x0(jj)>=0; dEx(jj,:)=k*tau*x1(jj,:)./r(jj,:).^3; dEy(jj,:)=k*tau./r(jj,:).^3;
    else; dEx(jj,:)=k*tau*x1(jj,:)./r(jj,:).^3; dEy(jj,:)=k*tau./r(jj,:).^3; end; end
Ex=squeeze(trapz(dEx,1)); Ey=Y.*squeeze(trapz(dEy,1));
F1=figure('units','normalized','outerposition',[0 0 1 1]); hax=axes; grid minor; hold all
hLines = streamslice(X,Y,Ex,Ey,5); set(hLines,'Color','r'); xlabel('-5 \leq X \leq 5','FontWeight','bold');
ylabel('-5 \leq Y \leq 5','FontWeight','bold'); plot(x0,0*x0,'k*'); axis tight
annotation(F1,'textarrow',[0.42 0.48],[0.42 0.51],'TextLineWidth',2,'FontSize',12,...
'String',{'Charged Line'},'HeadStyle','deltoid','LineWidth',2,'Color','k');

```

1. Describe how to estimate the relative strength and direction of E-field (see Section 1.6.3 in Chapter 1) using this plot. Mark area with the highest intensity of E-field.
2. Find the total flux of E- and D-fields flowing through the surface of the sphere surrounding the line of charges. Explain the results and compare them with the near-field E-field structure around an elementary electric dipole depicted in Figure 4.3.1 of Chapter 4.
3. If you like a challenge present all integrals in closed form. Then demonstrate analytically that the dominant components of dipole E-near-fields (see (5.20) and (5.21) in Chapter 5), and this charged line are almost identical, and both deviate as $\sim 1/r^2$ and $\sim 1/r^3$ nearby.
4. Show that the static and indeed nonradiating charged line is not able to create the E-far-field component $\sim 1/r$.
5. Ask the instructor for additional questions.

1.13E **E-fields of electrically short two-wire line.** Suppose a charge $+Q$ spreads uniformly along the x -axis with linear density $\tau = Q/L$ and a charge $-Q$ spreads uniformly along the same x -axis with linear density $\tau = -Q/L$, as shown here. $L \leq 1.5\text{m}$ is the total length each of charge lines and $d = L/8 \ll L$ (you can change it in Matlab script) is the separation between them. Sketch out the 2D equipotential lines of E-force produced by the charged lines. Then copy, paste, and run into Matlab Command Window the script slightly modified from 1.12E just to take into consideration the second charge line. Explain differences if you found them.



```

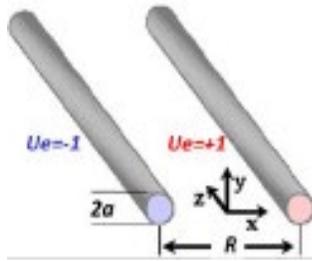
clc; close all; clear
display(' Length L of finite line of charge L<=1.5m'); L=input('Enter the length L [m] of finite line of charge: ');
d=input('Enter the separation d < L [m]: '); tau=input('Enter the linear density of charges [C/m]: ');
x=linspace(-L,L,100); y=linspace(-2*d,2*d,100); k = 8.987E9; [X,Y] = meshgrid(x,y); x0=linspace(-L/2,L/2,100);
for jj=1:size(x0,2)
    r1(jj,:)=sqrt((x0(jj)-X).^2+(Y-d/2).^2); xx(jj,:)=X-x0(jj);
    r2(jj,:)=sqrt((x0(jj)-X).^2+(Y+d/2).^2);
    dE1x(jj,:)=k*tau*xx(jj,:)./r1(jj,:).^3; dE1y(jj,:)=k*tau./r1(jj,:).^3;
    dE2x(jj,:)=k*tau*xx(jj,:)./r2(jj,:).^3; dE2y(jj,:)=k*tau./r2(jj,:).^3; end
dEx=dE1x+dE2x; Ex=squeeze(trapz(x,dEx,1)); Ey=(Y-d/2).*squeeze(trapz(y,dE1y,1))+(Y+d/2).*squeeze(trapz(y,dE2y,1));
figure('units','normalized','outerposition',[0 0 1 1]); hax=axes; grid minor; hold all;
hLines = streamslice(X,Y,Ex,Ey,10);set(hLines,'Color','r');
text(0,d/2+0.1,'+Q','Color','red','FontSize',50); text(0,-d/2-0.1,'-Q','Color','blue','FontSize',50)
xlabel('-1 \leq X \leq 1','FontWeight','bold'); ylabel('-1 \leq Y \leq 1','FontWeight','bold');
plot(x0,d/2,'k*');plot(x0,-d/2,'k*'); axis tight

```

1. Examine the E-field structure in the gap between the charge lines. Explain why it looks like E-field distribution (Section 6.1.1 in Chapter 6) along the length of the two-wire line.
2. Extrapolate this structure to the image of E-field between the capacitor plates shown in Figure 3.1.10b in Chapter 3. Draw several equipotential lines of potential.

- Find the total flux of E- and D-fields flowing through the surface of the sphere surrounding the lines of charges.
- Ask the instructor for additional questions.

1.14E E-field around Two Parallel Conductive Wires of Circular Cross-sections.



Suppose two straight conductive circular wires of identical diameter a and infinite length are surrounded by air. Their axes are separated by a distance R , parallel to the z -axis and symmetrically located relative to z - and x -axis. The battery (not shown) support equal and opposite potential of $1[V]$ on each wire. The following Matlab script calculates and pictures the potential and E-force lines in space outside wires. Sketch out the 2D equipotential lines of potential and E-force lines produced by these wires in the XY -plane. Then copy, paste, and run into Matlab Command Window. Explain differences if you found them.

```
clc; close all; clear; % http://bourabai.kz/assis/Revista-Bras-Ens-Fis-V21-p469-475\(1999\).pdf
display('Two Parallel Conductive Wires of Circular Cross-sections and Equal Radius')
a=input('Enter the radius a of wires [m]: ');
R=input('Enter the separation R > 2a (!) of wires [m]: ');
NN=400; x=linspace(-2*R,2*R,NN);y=linspace(-2*R,2*R,NN);
[X,Y] = meshgrid(x,y); Coeff=1/(2*log((R-sqrt(R^2-4*a^2))/(2*a)));
c1=(X-sqrt(R^2-4*a^2)/2).^2+Y.^2; c2=(X+sqrt(R^2-4*a^2)/2).^2+Y.^2;
Ue=Coeff*log(c1./c2);[Ex,Ey]=gradient(Ue,4*R/NN,4*R/NN); Umax=max(max(Ue));
Ue((X-R/2).^2+Y.^2<=a^2 | (X+R/2).^2+Y.^2<=a^2) = nan; C=8.854e-12*pi/acosh(R/(2*a));
F1=figure('units','normalized','outerposition',[0 0 1 1]); grid minor; hold all
hLines = streamslice(X,Y,-Ex,-Ey,4); set(hLines,'Color','r');
xlabel('X-axis [m]','FontWeight','bold'); ylabel('Y-axis [m]','FontWeight','bold');
th=0:pi/50:2*pi; x1=a*cos(th)+R/2; y1=a*sin(th); fill(x1,y1,'k');
x1=a*cos(th)-R/2+.005; y1=a*sin(th); fill(x1,y1,'k'); axis square
t1=text(-0.5*a,-1.2*a,'Wires'); set(t1,'FontSize',20,'FontWeight','Bold')
contour(X,Y,Ue,80); h1=colorbar('southoutside','fontsize',12); plot(0,-2*R:0.01:2*R,'k','LineWidth',1);
xlabel(h1,'Electric potential Ue(x,y) [V]','FontWeight','Bold','FontSize',15);
t2=text(-R/2-a,1.8*a,'Ue = -1V'); set(t2,'FontSize',20,'FontWeight','Bold','Color','b')
t3=text(R/2-a,1.8*a,'Ue = +1V'); set(t3,'FontSize',20,'FontWeight','Bold','Color','r')
xlabel('X-axis [m]','FontSize',15,'FontWeight','bold'); ylabel('Y-axis [m]','FontSize',15,'FontWeight','bold');
title(['Line Capacitance per meter = ',num2str(C),' [F/m]'],'Color','k','FontSize',20)
hold off; figure(2); h2=meshc(X,Y,Ue); g2=colorbar('fontsize',12); zlim([-Umax Umax]); axis tight
xlabel(g2,'Electric Potential Ue [V]','FontWeight','Bold','FontSize',15);
xlabel('x-axis','fontsize',14); ylabel('y-axis','fontsize',14); zlabel('Electric Potential [V]','fontsize',14)
h3=title('Electric Potential Distribution Ue(x,y) [V]'); set(h3,'fontsize',14); view(121,24); grid minor
vert = [0 -2*R -1;0 2*R -1;0 2*R -1;0 -2*R -1; 0 -2*R 1;0 2*R 1;0 2*R 1;0 -2*R 1];
fac = [1 2 6 5;1 2 6 5;1 2 6 5;1 2 6 5;1 2 6 5;1 2 6 5];
ph=patch('Vertices',vert,'Faces',fac,'FaceVertexCData',hsv(6),'FaceColor','flat');alpha(ph,0.2); view(-168,15)
```

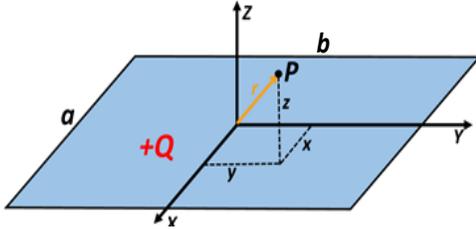
- Examine E-field structure. Explain why it is identical to shown in Figure 6.1.1 of Chapter 6.
- Please read Section 6.1.1 about TEM that explains this phenomenon. Describe how to estimate the relative strength and direction of E-field (see Section 1.6.3 in Chapter 1) using this plot. Mark area with the highest intensity of E-field.
- Could we expect and where Corona or air electrical breakdown as soon as the wire potential, as well as E-fields, become too high?
- Knowing that the normal component of E-field on the metal surface is equal to the density of electrical charges (read Section 2.3.2 in Chapter 2), sketch the qualitative profile of these charges on the wire surfaces.
- Explain why no electric field may exist inside a conductor (read Section 2.2.7 in Chapter 2).
- Check on the graph that tangential component of E-field is really zero there (see Figure 2.3.5 in Chapter 2).
- Find the total flux of E- and D-fields flowing through the surface of the cylinder surrounding the two-wire line.
- Evaluate the potential value along the symmetry line (black dotted) in the first figure and pink symmetry plane in the second. How much is this potential and why? What happens if we put an infinite conductive plate

of an infinitesimal thickness in this plane? Note that this slightly upgraded model can be used to estimate E-fields and potential underneath overhead high voltage power line. The excessive level of them is significant public concern about adverse health effect.

9. Ask the instructor for additional questions.

More details can be found in [http://bourabai.kz/assis/Revista-Bras-Ens-Fis-V21-p469-475\(1999\).pdf](http://bourabai.kz/assis/Revista-Bras-Ens-Fis-V21-p469-475(1999).pdf)

1.15E **Uniformly Charged Sheet.** Consider a uniformly charged finite rectangular ($a \times b$) sheet carrying a positive charge Q that spreads uniformly over the area with a surface density $\rho_{eA} = Q/ab$ [C/m²]. Sketch out the equipotential lines of potential and E-force on some plane above the sheet ($Z = \text{const.}$). Then copy, paste, and run the following Matlab script that calculates and plots the lines of equal potential and E-force on the plane $Z \leq 10\text{m}$, i.e. above the sheet, starting, for example, from $Z = 0.1\text{m}$. Explain differences if you found them.



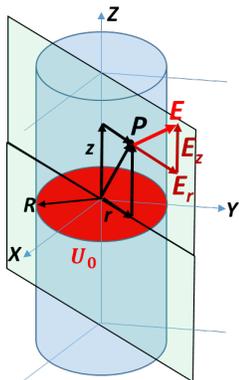
```

clc; clear;
a=input('Enter the length of square side a<=1m: '); b=input('Enter the length of square side b<=1m: ');
Z=input('Field plane location Z<=10m: '); sum=0;N=121; NN=250;
x=linspace(-5*a,5*a,NN); y=linspace(-5*b,5*b,NN); [X,Y]=meshgrid(x,y);
for jx=1:N; x0=-a/2+(jx-1)*a/(N-1); for jy=1:N; y0=-b/2+(jy-1)*b/(N-1);
R=sqrt((X-x0).^2+(Y-y0).^2+Z^2); Phi=1./R; sum=sum+Phi;
end; end; sum=sum/max(sum); [Ex,Ey]=gradient(sum,a/NN,b/NN);
F1=figure; grid minor; hold on; hLines = streamslice(X,Y,-Ex,-Ey,2);set(hLines,'Color','r','LineWidth',2);
contour(X,Y,sum,20,'LineWidth',2); E=sqrt(Ex.^2+Ey.^2); C=del2(E);
s=meshc(X,Y,-E-Z); zticks([0 Z]); zticklabels({'Z = ',num2str(Z)}); view([53,24]); alpha(s,0.1);
xlabel('X-axis [m]'); ylabel('Y-axis [m]'); zlabel('Z-axis [m]');
annotation(F1,'textarrow',[0.52 0.48],[0.45 0.34],'TextLineWidth',2,'FontSize',18,...
'String', {'|E(x,y)| at Z = ',num2str(Z),'m'},'HeadStyle','deltoid','LineWidth',2,'Color','r');

```

1. After the plot appears, retain it on the computer screen by moving it slightly from the center. Then paste the script into Matlab Command Window again and change the plane height to $Z = 1\text{m}$, for example, keeping a and b unaffected. Repeat this procedure several times up to $Z = 10\text{m}$. Compare all plots and clarify the potential and E-field variations in the set.
2. How to explain the persistent peak of potential and E-field intensity at the coordinate origin?
3. What is the orientation of E-fields along the z -axis?
4. Assume that $Z \rightarrow \infty$. Explain why the E-field looks like the E-field of a point charge and $|E| \sim 1/r^2$ where r is the far-away distance from the sheet.
5. Using Gauss's Law (expression (1.23) in Chapter 1) determine and plot the E-field intensity at some distance r in front of an infinite sheet ($a \rightarrow \infty, b \rightarrow \infty$) carrying a uniform surface charge.
6. Ask the instructor for additional questions.

1.16E **Uniformly Charged Disk.** Find the E-field and potential at point P located in the rZ plane due to a **nonconductive** disk carrying the uniformly distributed charge Q^3 . The disk of zero thickness has the radius R . The rotation symmetry means that the E-field image is reproduced identically in any containing z -axis vertical plane (illustrated in light green). Sketch out the 2D equipotential lines of potential and E-force produced by the disk in plane rZ . Then copy, paste, and run the following Matlab script that calculates and plots the lines of equal potential and E-force. Explain differences if you found them.



1. Describe how to estimate the relative strength and direction of E-field (see Section 1.6.3 in Chapter 1) using this plot. Mark area with the highest intensity of E-field.

³ The expressions behind this script can be found on

<https://www.sciencedirect.com/science/article/pii/S0893965911002564#br000040>

- Find the total flux of E- and D-fields flowing through the surface of the sphere surrounding the charged disk.
- Ask the instructor for additional questions.

```

clc; close all; clear; % Analytical solution
R=input('Enter the Disk Radius 0 < R [m] < 2: ');
Q=input('Enter the disk Charge Q [C] = ');
k=8.9876e+09; Coeff=2*k*Q/R; N=100;
rho=linspace(-2*R,2*R,N); z=linspace(-2*R,2*R,N); [Rho,Z]=meshgrid(rho,z);
for i=1:N; for j=1:N;
fun = @(k0) besselj(0,k0.*Rho(i,j)).*(besselj(1,k0.*R)./k0).*exp(-k0.*abs(Z(i,j)));
U(i,j)=integral(fun,0,inf); end; end; Ue=Coeff*U; [Er,Ez]=gradient(Ue,4*R/N,4*R/N);
f3=figure('units','normalized','outerposition',[0 0 1 1]); grid minor; hold on
hLines = streamslice(Rho,Z,Er,Ez,2);set(hLines,'Color','r','LineWidth',1);
hContour = contour(Rho,Z,Ue,40); hColorbar = colorbar;
ylabel(hColorbar,'Electric potential Ue [V]','FontWeight','Bold','FontSize',12);
xlabel('r-axis [m]'); ylabel('z-axis [m]'); r0=linspace(-R,R,200); plot(r0,0*r0,'k'); axis tight; axis square
annotation(f3,'textarrow',[0.42 0.48],[0.42 0.51],'TextLineWidth',2,'FontSize',20, ...
'String',{'Uniformly Charged Disk'},'HeadStyle','deltoid','LineWidth',2,'Color','k'); hold off;
figure(2);s=mesh(Rho,Z,Ue); xlabel('r-axis','FontSize',14,'FontWeight','bold');
ylabel('z-axis','FontSize',14,'FontWeight','bold'); zlabel('Potential Ue(r,z)','FontSize',14,'FontWeight','bold')
title('Electric Potential Distribution in rz-plane','FontSize',12,'FontWeight','bold')

```

1.17E **The static model of a capacitive top-hat element of the current radiator** (see Figure 4.2.1 in Chapter 4). Find the E-field and potential at point P located in the rZ plane due to a perfectly **conducting** disk (grasp the image above) at *absolute* potential $U_0 = 1$ (see Section 1.6.2 in Chapter 1 and Section 2.2.7 in Chapter 3). The disk of zero thickness has the radius R . The rotation symmetry means that the E-field image is reproduced identically in any containing z -axis vertical plane (illustrated in light green). Sketch out the 2D equipotential lines of potential and E-force produced by the disk in plane rZ . Then copy, paste, and run the following Matlab script that calculates and plots the lines of equal potential and E-force (shown in red)⁴.

```

clc; close all; clear; %Analytical Solution of Laplace Equation in the oblate spheroidal coordinate
R=input('Enter the Disk Radius 0 < R [m] < 2: '); N=120;
theta=linspace(0,pi/2,N); eta=linspace(1e-2,log(acosh(4)),N);
[Eta,Theta]=meshgrid(eta,theta); r=R*cosh(Eta).*sin(Theta); z=R*sinh(Eta).*cos(Theta);
Ue=-(-imag(acsc(sinh(Eta))))); U0=max(max(abs(Ue))); Ue=Ue/U0;
E1=gradient(Ue); Ez=E1.*cosh(Eta).*cos(Theta); Er=E1.*sinh(Eta).*sin(Theta); C=8*8.8542e-12*R;
f1=figure('units','normalized','outerposition',[0 0 1 1]); grid minor; hold on
contour(r,z,Ue,40); contour(-r,z,Ue,40); contour(r,-z,Ue,40); contour(-r,-z,Ue,40);
hColorbar = colorbar; ylabel(hColorbar,'Electric potential Ue','FontWeight','Bold','FontSize',12);
xlabel('r-axis [m]'); ylabel('z-axis [m]'); r0=linspace(-R,R,200); plot(r0,0*r0,'k'); axis tight;
annotation(f1,'textarrow',[0.42 0.48],[0.42 0.51],'TextLineWidth',2,'FontSize',20, ...
'String',{'Conducting Disk'},'HeadStyle','deltoid','LineWidth',2,'Color','k'); axis square
text(-R,max(max(z))/2,['Disk Capacitance = ',num2str(C),'[F]'],'Color','k','FontSize',24)
Lines = quiver(r,z,-Er,-Ez,2);set(Lines,'Color','r','LineWidth',1);
Lines = quiver(-r,z,Er,-Ez,2);set(Lines,'Color','r','LineWidth',1);
Lines = quiver(r,-z,-Er,Ez,2);set(Lines,'Color','r','LineWidth',1);
Lines = quiver(-r,-z,Er,Ez,2);set(Lines,'Color','r','LineWidth',1); hold off;
figure(2);hold on; s=meshc(r,z,Ue); s1=meshc(-r,z,Ue); xlabel('r-axis','FontSize',14,'FontWeight','bold');
hColorbar = colorbar; view(-23,10); ylabel('z-axis','FontSize',14,'FontWeight','bold'); grid minor
zlabel('Potential Ue(r,z)','FontSize',14,'FontWeight','bold')
title('Electric Potential Distribution in rz-plane','FontSize',12,'FontWeight','bold');

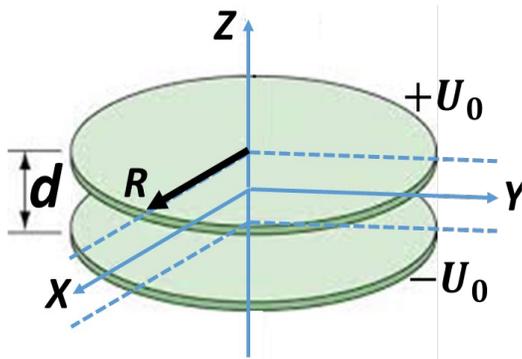
```

- Explain differences if you found them. Describe the impact of the conductive disk on the potential and E-field patterns making them different from the plotted ones in problem 1.6.11.
- Why a conductive surface is always equipotential in static fields?
- Explain the fringing effect, i.e. the bending and high concentration of the E-force lines near the edge of the disk.

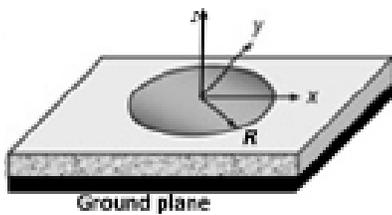
⁴ The expressions behind this script can be found on <http://physics.usask.ca/~hirose/p812/notes/Ch2.pdf>

4. Check the boundary conditions in Section 2.3.5 of Chapter 2 and sketch the free charge ρ_{eA} distribution on the disk surface and make sure that it is not uniform as it was assumed in the problem 1.16E.
5. Could we expect and where Corona or air electrical breakdown as soon as the disk potential, as well as E-fields, become too high?
6. May you offer the disk design with the edge effect reduction after reading the information in Section 3.3.3 of Chapter 3? Look at the practical solution in https://quickfield.com/advanced/rogowski_electrodes.htm
7. On the top of the E-field plot, you can find the disk capacitance calculated using the simulation data and expressions (3.25), (3.26) from Chapter 3. How does this capacitance change if the fringe fields are omitted?
8. Read Sections 3.3.2 and 3.3.3 in Chapter 3 for more details.
9. Ask the instructor for additional questions.

1.18E **Capacitor with circular plates.** A capacitor comprises two parallel conductive disks of the same radius R equally and oppositely charged to *absolute* potentials $+U_0$ and $-U_0$. They are of an infinitesimal thickness and placed a distance d apart. The rotation symmetry means that the E-field structure is reproduced identically in any containing z -axis vertical plane. Sketch out the 2D equipotential lines of potential and E-force produced by the disks in one of this plane. Then copy, paste, and run the following Matlab script⁵ (see next page). Figure 1 demonstrates the electric potential U_e as the equipotential contour curves and E-force as the red arrowed lines. Figure 2 is a 3D image of the potential distribution. Explain differences if you found them.



1. How to clarify the potential and E-field deviations nearby the plate edges and its almost uniform distribution between the plates?
2. Comment the reason of E-field singularity (enormous increase E-field) around the edges (check Section 3.3.2 in Chapter 3 and Table 3.1 there).
3. Evaluate the potential value along the symmetry line (black dotted) in the first figure and pink symmetry plane in the second. How much is this potential and why?



4. What happens if we put an infinite conductive plate of an infinitesimal thickness in this plane?
5. Draw a 3D image of the disk above this ground plane and get the static model of the capacitor in the printed circuit board (PCB) or electrically small patch antenna shown on the left.
6. Run the script for $d/R \geq 2$ and comment results. Compare electric potential and E-force line structure with the structure generated by two point charges in problem 1.6E.
7. Explain the difference between two values of capacitance printed on the top of Figure 1.
8. Ask the instructor for additional questions.

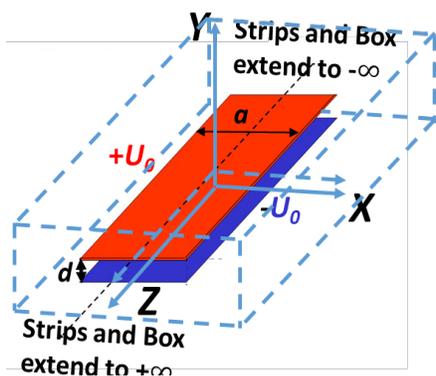
⁵ Great gratitude to Dr. Gradimir V. Milovanovic for giving permission to use the results of the publication <http://www.mi.sanu.ac.rs/~gvm/radovi/gvm-dule1.pdf> and providing additional data included in the script.

```

clc; close all; clear; fprintf(2,'\nAll coordinates are normalized to disk radius R\n');
fprintf(2,'\nThe normalized to R Gap d/R between disks must be chosen as\n');
fprintf(2,'\nd/R = 0.1 - 1 with step 0.1 or d/R = 2,5,10\n'); d=input('\nEnter the ratio d/R = ');
NN=1.2e2; R=1; r= linspace(0,2*R,NN); z= linspace(-4*d,4*d,NN); [RR,ZZ] = meshgrid(r,z);
if d==0.1; P=@(x) (11.170089-4.690523*x.^2-1.474409*x.^4+0.664140*x.^6-1.53822237*x.^8); end
if d==0.2; P=@(x) (6.0773646-2.373046*x.^2+0.164289*x.^4-1.6199941*x.^6+0.7925024*x.^8); end
if d==0.3; P=@(x) (4.3596859-1.507951*x.^2-0.002886*x.^4-0.7929827*x.^6+0.5076101*x.^8); end
if d==0.4; P=@(x) (3.4935288-1.072782*x.^2-0.118818*x.^4-0.2410555*x.^6+0.2211096*x.^8); end
if d==0.5; P=@(x) (2.97067-0.819293*x.^2-0.124922*x.^4-0.0114976*x.^6+0.077654494*x.^8); end
if d==0.6; P=@(x) (2.6206072-0.653164*x.^2-0.088666*x.^4+0.0576722*x.^6+0.0184320*x.^8); end
if d==0.7; P=@(x) (2.3699300-0.533814*x.^2-0.048402*x.^4+0.0639551*x.^6-0.0022945*x.^8); end
if d==0.8; P=@(x) (2.1817877-0.442585*x.^2-0.017287*x.^4+0.0511089*x.^6-0.0075254*x.^8); end
if d==0.9; P=@(x) (2.0356215-0.370213*x.^2+0.002996*x.^4+0.0358529*x.^6-0.0073136*x.^8); end
if d==1.0; P=@(x) (1.91903-0.311595*x.^2+0.014564*x.^4+0.0233527*x.^6-0.005655492*x.^8); end
if d==2.0; P=@(x) (1.41007-0.071116*x.^2+0.008674*x.^4-0.0005066*x.^6-0.0000309361*x.^8); end
if d==5; P=@(x) (1.14348-0.005373*x.^2+0.000182*x.^4); end
if d==10; P=@(x) (1.06773-0.000666*x.^2+0.000000*x.^4); end; for ii=1:length(r); for jj=1:length(z);
VV=@(x) (1./sqrt(RR(ii,jj)^2+(ZZ(ii,jj)-d/2+1i*x).^2))-1./sqrt(RR(ii,jj)^2+(ZZ(ii,jj)+d/2+1i*x).^2)).*P(x);
Int(ii,jj)=integral(VV,-1,1); end; end; Ue=real(Int)/pi; [Er Ez]=gradient(Ue,2*R/NN,8*d/NN);
C=8.854e-12*sum(sum(Er.^2+Ez.^2))*(2*R/NN)*(8*d/NN); C0=8.854e-12*pi*R^2/d;
figure('units','normalized','outerposition',[0 0 1 1]); grid minor; hold on;
hLines = streamslice(RR,ZZ,-Er,-Ez,2);set(hLines,'Color','r','LineWidth',1);
hLines = streamslice(-RR,ZZ,Er,-Ez,2);set(hLines,'Color','r','LineWidth',1);
contour(RR,ZZ,Ue,50); contour(-RR,ZZ,Ue,50); hh=colorbar;
ylabel(hh,'Electric Potential Intensity Ue(r/R,z/R)','FontWeight','Bold','FontSize',15);
xlabel('r/R-axis [m]'); ylabel('z/R-axis [m]'); r0=linspace(-R,R,200);
axis 'tight'; rectangle('Position',[-R,-d/2,2*R,4*d/NN],'FaceColor','k');
rectangle('Position',[-R,d/2,2*R,4*d/NN],'FaceColor','k');
text(0,-d/2,'Uo = -1','Color','b','FontSize',50); text(0,1.1*d/2,'Uo = +1','Color','r','FontSize',50)
text(-2*R,4*d,['Capacitance according to (3.26) = ',num2str(C0),' [F]'],'Color','k','FontSize',40)
text(-2*R,3.4*d,['Capacitance according to (3.27) = ',num2str(C),' [F]'],'Color','k','FontSize',40);
if d<2; plot(-2*R:0.1:2*R,0,'*m','LineWidth',4); end; if d>=2; axis square; xlim([-4*d,4*d]); end;
figure(2);hold on; meshc(RR,ZZ,Ue); meshc(-RR,ZZ,Ue); xlabel('r/R-axis','FontSize',14,'FontWeight','bold');
ylabel('z/R-axis','FontSize',14,'FontWeight','bold'); view(84,21);
hh1=colorbar; ylabel(hh1,'Electric Potential Intensity Ue(r/R,z/R)','FontWeight','Bold','FontSize',15);
zlabel('Electric Potential Ue(r/R,z/R)','FontSize',14,'FontWeight','bold');grid minor;
title('Electric Potential Intensity in rz-plane','FontSize',12,'FontWeight','bold')
vert = [-2*R 0 -1;2*R 0 -1;2*R 0 -1;-2*R 0 -1;-2*R 0 1;2*R 0 1;2*R 0 1;-2*R 0 1];
fac = [1 2 6 5;1 2 6 5;1 2 6 5;1 2 6 5;1 2 6 5;1 2 6 5];
ph=patch('Vertices',vert,'Faces',fac,'FaceVertexCData',hsv(6),'FaceColor','flat');alpha(ph,0.2)

```

1.19E 2D Parallel Strip model of Capacitor. A capacitor comprises two parallel conductive strips of width a equally and oppositely charged to potentials $+U_0$ and $-U_0$ as $U_0 = 100\text{V}$. They are of an infinitesimal thickness and placed a distance $d = 0.6a$ apart. Generally $d \ll a$, but accepting such practically rear ratio as $d/a = 0.6$ allows to demonstrate noticeably the effect of fringe fields (look at Figure 3.1.10 in Chapter 1). The strips are placed into a relatively large metal box to make the numerical simulation manageable short. This code solves the 2D Poisson's equation numerically for electric potential (for more details read



https://www.mathworks.com/matlabcentral/fileexchange/42773-electric-field-of-a-parallel-plate-capacitor-using-2d-poisson-equation). Sketch out your version of 2D equipotential lines of potential and E-force produced by the strips at XY-plane. Then copy, paste, and run the following Matlab script. The first appeared figure demonstrates the electric potential Ue as the equipotential contour curves and E-force as the red arrowed lines. The second figure is the contour plot of the E-field magnitude, while the third one is a 3D image of the potential distribution.

<https://www.mathworks.com/matlabcentral/fileexchange/42773-electric-field-of-a-parallel-plate-capacitor-using-2d-poisson-equation>

The first appeared figure demonstrates the electric potential Ue as the equipotential contour curves and E-force as the red arrowed lines. The second figure is the contour plot of the E-field magnitude, while the third one is a 3D image of the potential distribution.

```

clc; close all; clear; % Poisson's Equation numerical solution
a=input('Input the strip width a<=5m: '); d=input('Input the strip separation d<a: ');
Nx = 101; Ny = 121; mpx=ceil(Nx/2); mpy=ceil(Ny/2); Ni = 750; h=d/a;
V = zeros(Nx,Ny); T = 0; B = 0; L = 0; R = 0; V(1,:) = L; V(Nx,:) = R; V(:,1) = B; V(:,Ny) = T;
V(1,1) = 0.5*(V(1,2)+V(2,1)); V(Nx,1) = 0.5*(V(Nx-1,1)+V(Nx,2));
V(1,Ny) = 0.5*(V(1,Ny-1)+V(2,Ny)); V(Nx,Ny) = 0.5*(V(Nx,Ny-1)+V(Nx-1,Ny));
l_p = 51; lp = floor(l_p/2);
p_p = ceil(l_p*h/2); pp1 = mpx+p_p; pp2 = mpx-p_p;
for z = 1:Ni; for i=2:Nx-1; for j=2:Ny-1; V(pp1,mpy-lp:mpy+lp) = 100; V(pp2,mpy-lp:mpy+lp) = -100;
V(i,j)=0.25*(V(i+1,j)+V(i-1,j))+V(i,j+1)+V(i,j-1)); end; end; end
V = V'; [Ex,Ey]=gradient(V); Ex=-Ex; Ey=-Ey; E=sqrt(Ex.^2+Ey.^2); x=(1:Nx)-mpx; y=(1:Ny)-mpy;
% Display Potential and E-force Lines
figure('units','normalized','outerposition',[0 0 1 1]);grid minor; hold on;
hLines = streamslice(x,y,Ex,Ey,4);set(hLines,'Color','r','LineWidth',1);
contour(x,y,V,60); h1=colorbar('southoutside','fontsize',12);
xlabel(h1,'Electric potential Ue [V]','FontWeight','Bold','FontSize',20);
title('Electric Potential Ue(x,y)and E-force Lines','fontsize',14); xlabel('x-axis','fontsize',14); ylabel('y-axis','fontsize',14);
rectangle('Position',[-p_p-0.5,-l_p/2,1,l_p],'FaceColor','k')
rectangle('Position',[p_p,-l_p/2,1,l_p],'FaceColor','k'); plot(0,-(Ny-1)/2:1:(Ny-1)/2,'*k');
% Contour Display of E-field
f3=figure(2); contour(x,y,E,120); g2=colorbar('fontsize',12);
ylabel(g2,'E-field Intensity [V/m]','FontWeight','Bold','FontSize',15);
xlabel('x-axis','fontsize',14); ylabel('y-axis','fontsize',14);
h2=title('Electric field distribution, E (x,y) in V/m','fontsize',14);
set(h2,'fontsize',14); movegui(f3,[100,100]); grid minor
% Contour Display of electric potential
figure(3); hh=meshc(x,y,V); axis([-2*lp,2*lp,-2*lp,2*lp]); g3=colorbar('southoutside','fontsize',12);
xlabel(g3,'Electric Potential Ue [V]','FontWeight','Bold','FontSize',15);
xlabel('x-axis','fontsize',14); ylabel('y-axis','fontsize',14); zlabel('Electric Potential [V]','fontsize',14)
h3=title('Electric Potential Distribution, V(x,y) [V]'); set(h3,'fontsize',14); view(-13,32); grid minor
set(hh(2),'ShowText','on','LineWidth',1);
vert = [0 -50 -100;0 50 -100;0 50 -100;0 -50 -100; 0 -50 100;0 50 100;0 50 100;0 -50 100];
fac = [1 2 6 5;1 2 6 5;1 2 6 5;1 2 6 5;1 2 6 5;1 2 6 5];
ph=patch('Vertices',vert,'Faces',fac,'FaceVertexCData',hsv(6),'FaceColor','flat');alpha(ph,0.2)

```

1. Explain differences if you found them.
2. How to describe the potential and E-field deviations nearby the plate edges and its almost uniform distribution between the plates?
3. Comment the reason of E-field singularity (colossal increase E-field) around the edges (check Section 3.3.2 in Chapter 3 and Table 3.1 there).
4. Evaluate the potential value along the symmetry line (black dotted) in the first figure and pink symmetry plane in the second. How much is this potential and why?
5. What happens if we put an infinite conductive plate of an infinitesimal thickness in this plane?
6. Compare the picture in Figure 6.1.1c of Chapter 6 and explain why the E-field pattern in microstrip line shielded by a metal box looks so similar (read Section 6.1.1 about TEM mode).
7. Ask the instructor for additional questions.