

To boost electromagnetics' flavor, we accompanied many problems with short Matlab code acting as a *calculator and result illustrator*. We completely ignored the description of algorithms behind the codes because they are often quite complicated and required ample, very sophisticated, abstract mathematical specifics to help understand the physical picture. Such an approach shifts the focus from the often emasculated and practically fruitless problems primarily based on math transformations to more complicated but close to practical tasks. We hope to visualize typically invisible EM fields, stimulate in-depth analysis, and discuss the fundamental principles. Projecting images onto a big screen, the lectures may organize whole-class conversation making the audience significantly engaged.

We cheer our readers to look through Matlab scripts to discover and sophisticate the algorithm embedded in them. We encourage you to use the student edition of MATLAB and CST STUDIO SUITE® to get enhanced problem understanding. Sorry for the compact transcription of scripts that is dictated by the blog environment. **A better readable version of scripts will be sent to you upon request left on the Contact page.**

Attention. Regrettably, copy and paste into Matlab Command Window saves appropriate Matlab format just in Chrome Web Browser. You have to restart <https://emfieldbook.com/> in <https://www.google.com/> if your browser is different.

Problem 1. Dimensional Analysis and Unit Law

1.1. Use Table 1.5 to check which of the following equations is dimensionally valid and describes Lorentz's force equation correctly

- a) $\mathbf{F}_{em} = Q_e(\mathbf{E} + \mathbf{v} \times \mathbf{B})$ b) $\mathbf{F}_{em} = Q_e(\mathbf{D} + \mathbf{v} \times \mathbf{H})$
 c) $\mathbf{F}_{em} = Q_m(\mathbf{E} + \mathbf{v} \times \mathbf{B})$ d) $\mathbf{F}_{em} = Q_e\mathbf{E} + Q_m\mathbf{v} \times \mathbf{B}$

1.2. Use Table 1.5 to check which of the following equations is dimensionally valid and describes the 1st Maxwell's equation in the differential form correctly

- a) $-\nabla \times \mathbf{E} = \frac{\partial \mathbf{H}}{\partial t} + \mathbf{j}_{mV}$ c) $-\nabla \times \mathbf{E} = \frac{\partial \mathbf{B}}{\partial t} + \mathbf{j}_{eV}$
 b) $-\nabla \times \mathbf{E} = \frac{\partial \mathbf{B}}{\partial t} + \mathbf{j}_{mV}$ d) $-\nabla \times \mathbf{D} = \frac{\partial \mathbf{H}}{\partial t} + \mathbf{j}_{mV}$

1.3. Use Table 1.5 to check which of the following equations is dimensionally valid and describes the 2nd Maxwell's equation in the differential form correctly

- a) $\nabla \times \mathbf{H} = \frac{\partial \mathbf{E}}{\partial t} + \mathbf{j}_{eV}$ c) $\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{j}_{mV}$
 b) $\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{j}_{eV}$ d) $\nabla \times \mathbf{B} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{j}_{eV}$

1.4. Use Table 1.5 to check which of the following equations is dimensionally valid and describes the *electric* charge conservation law correctly

- a) $\oint\!\!\!\oint_A \mathbf{j}_{mV} \circ d\mathbf{A} + \frac{dQ_e(t)}{dt} = 0$ c) $\oint\!\!\!\oint_A \mathbf{j}_{eV} \circ d\mathbf{A} + \frac{dQ_m(t)}{dt} = 0$
 b) $\oint_L \mathbf{j}_{eV} \circ d\mathbf{A} + \frac{dQ_e(t)}{dt} = 0$ d) $\oint\!\!\!\oint_A \mathbf{j}_{eV} \circ d\mathbf{A} + \frac{dQ_e(t)}{dt} = 0$

1.5. The vector \mathbf{H} of the *static* magnetic field is measured in base units [A/m] along with some closed path L . Use Table 1.5 to check which of the following equations is dimensionally valid and describes Ampere's Law correctly

- a) $\oint_L \mathbf{H} \circ d\mathbf{L} = I_m$ c) $\oint_L \mathbf{H} \circ d\mathbf{L} = Q_e$

$$b) \oint_L \mathbf{H} \circ d\mathbf{L} = I_e$$

$$d) \oint_L \mathbf{H} \circ d\mathbf{L} = Q_m$$

1.6. The vector \mathbf{D} of the electric displacement field is measured in base units [As/m²] along some closed surface A . Use Table 1.5 to check which of the following equations is dimensionally valid and describes the 3rd Maxwell's equation (Gauss's Law) correctly

$$a) \oiint_A \mathbf{D} \circ d\mathbf{A} = I_e$$

$$c) \oiint_A \mathbf{D} \circ d\mathbf{A} = I_m$$

$$b) \oiint_A \mathbf{D} \circ d\mathbf{A} = Q_e$$

$$d) \oiint_A \mathbf{D} \circ d\mathbf{A} = Q_m$$

1.7. Vector \mathbf{B} of the magnetic inductance field is measured in base units [As/m²] on some closed surface A . Use Table 1.5 to examine which of the following equations is dimensionally valid and describes the 3rd Maxwell's equation (Gauss's Law) correctly

$$a) \oiint_A \mathbf{D} \circ d\mathbf{A} = I_e$$

$$c) \oint_L \mathbf{D} \circ d\mathbf{L} = I_e$$

$$b) \oiint_A \mathbf{D} \circ d\mathbf{A} = Q_e$$

$$d) \oiint_A \mathbf{D} \circ d\mathbf{A} = Q_m$$

1.8. You found reading several publications that power P_Σ emitted by the infinitesimal current element is defined as

$$a) P_\Sigma = \frac{8\pi}{3} \left(\frac{I_e^{exc} \Delta l}{2\sqrt{2}\lambda} \right)^2 / Z_0$$

$$c) P_\Sigma = \frac{8\pi}{3} \left(\frac{I_e^{exc} \Delta l}{2\sqrt{2}\lambda} \right)^2 Z_0$$

$$b) P_\Sigma = \frac{8\pi}{3} \left(\frac{I_e^{exc} \Delta l}{2\sqrt{2}\lambda} \right) Z_0^2$$

$$d) P_\Sigma = \frac{8\pi}{3} \left(\frac{I_e^{exc} \Delta l}{2\sqrt{2}\lambda} \right)^2 Z_0^2$$

Which expression may be correct if Δl is the physical length of the element, I_e^{exc} is the electric current, λ is a wavelength in [m], and Z_0 is the impedance of free space in [Ohms] (check Section 4.2 in Chapter 4 for details)?

1.9. An inductance L [H] is attached in parallel or series to a capacitance C [F]. Using data from Table 1.5, find such a combination of these values that correctly defines the angular resonance frequency ω [1/s] of such a parallel resonance circuit.

1.10. The distributed parameters of some transmission line are \mathcal{L}' [H/m] and \mathcal{C}' [F/m] (see Sections 6.1 and 6.5 of Chapter 6 for details). Using data from Table 1.5, find such a combination of these parameters that correctly defines the characteristic line impedance Z_c [Ohms].

1.11. Using the first Maxwell's equation in Table 1.7 of Chapter 1 and knowing that \mathbf{E} [V/m] find the dimension unit of magnetic current I_m . Are you surprised? Read Section 1.4.4 and explain the result. Check Law of electromagnetic induction (1.42) for better understanding.

APPENDIX. Vector Differential Operators in Cartesian, Cylindrical, and Spherical Coordinate System.

The following Matlab scripts return analytical expressions of the gradient of scalar function and divergence and curl of vector-function. A wide variety of mathematical functions and their combinations, including special functions like Bessel, Legendre, Chebyshev, and many others,¹ are accepted. The only requirement is

¹ Look through Matlab documentation or the list in Symbolic Math Tool – Functions at https://www.mathworks.com/help/symbolic/referencelist.html?searchHighlight=symbolic%20legendre&s_tid=doc_srchtile&type=function

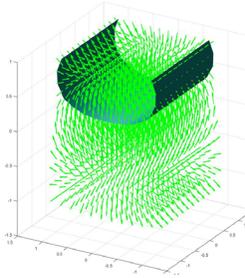
to use the symbolic sentence structure that is recognizable in Matlab Symbolic Toolbox. For example, $(1, r) * \text{legendreP}(1, \cos(\theta)) * \sin(\phi)$ means the modified Bessel function of the first kind times Legendre polynomial times cosine function. These scripts make you very forceful to get and check practically any equation in this and many other books of electrodynamics, antennas, microwave, etc. For example, you can apply analytically and get without errors (!) the vector operator curl ($\text{curl } \mathbf{F}$) in a spherical coordinate system. A relatively limited number of humans might perform such an act of bravery. Please look at expression (A.46) on page 473 and correct its left side as $\text{grad}(\text{div } \mathbf{F}) - \nabla^2 \mathbf{F}$. Sorry for the typo.

Problem 2. The Gradient of Scalar Function. The local function in the Matlab script below is taken from

https://www.mathworks.com/matlabcentral/fileexchange/66482-gradient_sym-v-x-coordinate_system. **Warning!** Copy and paste Matlab script below into a newly created file in **Editor Window**, then save it as GradF.m, for example. The presence

of local functions in the script requires Matlab R2016b or later. Push Run button  in Editor Window and proceed with simulation following the instructions on the screen. By definition, the numerical simulation of gradient takes place inside a cube's surface $\max\{|x|, |y|, |z|\} = 1$. You can adjust the Matlab script if you wish to change it. *Be careful with Matlab syntax entering math expressions!* Choose the Cartesian

coordinate system first and run the example depicted on the screen. The force lines of gradient vector $\text{grad } F$ (shown as green cones) cross the isosurfaces (dark green) where $F = \text{const}$. For the sake of better presentation, this constant was chosen 0.5 while $\text{abs}(F) \leq 1$. You can adjust the Matlab script if you wish to change it. Pay attention and then explain the mutual orientation of the vector fields and isosurfaces.



```
close all; clc; clear;
n=input('Enter 1 for Cartesian, 2 for Cylindrical, and 3 for Spherical: ');
disp('Follow Matlab rules for math expressions. ');
N=16; [Xq,Yq,Zq]=meshgrid(linspace(-1,1,N)); syms x y z r theta phi
switch n
case 1
    F=input(['Enter scalar fuction of x, y, z (lowercase letters only!) as [Fx, Fy, Fz].!...
            '\nFor example, cos(pi*y/3)*sin(2*pi*z/3) ']);
    C=gradient_Sym(F,[x,y,z],'Cartesian'); C0=formula(C);
    C1=double(subs(C0(1),[x,y,z],[Xq,Yq,Zq])); C2=double(subs(C0(2),[x,y,z],[Xq,Yq,Zq]));
    C3=double(subs(C0(3),[x,y,z],[Xq,Yq,Zq])); F1=double(subs(F,[x,y,z],[Xq,Yq,Zq]));
case 2
    F=input(['\nEnter scalar fuction of r, phi, z (lowercase letters only!) as [Fr, Fphi, Fz].!...
            '\nFor example, r*sin(phi) ']);
    R=sqrt(Xq.^2+Yq.^2+Zq.^2); Phi=atan2(Yq,Xq);
    C=gradient_Sym(F,[r,phi,z],'Cylindrical'); C0=formula(C);
    C1=double(subs(C0(1),[r,phi,z],[R,Phi,Zq])); C2=double(subs(C0(2),[r,phi,z],[R,Phi,Zq]));
    C3=double(subs(C0(3),[r,phi,z],[R,Phi,Zq])); F1=double(subs(F,[r,phi,z],[R,Phi,Zq]));
case 3
    F=input(['\nEnter scalar fuction of r, theta, phi (lowercase letters only!) as [Fr, Ftheta, Fphi].!...
            '\nFor example, r*cos(theta)*sin(phi) ']);
    R=sqrt(Xq.^2+Yq.^2+Zq.^2); Theta=acos(Zq./R); Phi=atan2(Yq,Xq);
    C=gradient_Sym(F,[r,theta,phi],'Spherical'); C0=formula(C);
    C1=double(subs(C0(1),[r,theta,phi],[R,Theta,Phi])); C2=double(subs(C0(2),[r,theta,phi],[R,Theta,Phi]));
    C3=double(subs(C0(3),[r,theta,phi],[R,Theta,Phi])); F1=double(subs(F,[r,theta,phi],[R,Theta,Phi]));
end; fprintf(2, 'The scalar function is:\n'); F
fprintf(2, 'The gradient vector is:\n'); Grad_F=C'
F1=F1/max(abs(F1(:))); figure('units','normalized','outerposition',[0 0 1 1]); grid on, hold on; axis tight
h1=isosurface(Xq,Yq,Zq,F1,0.5,F1); p1=patch(h1); set(p1,'FaceColor','blue','EdgeColor','none','FaceAlpha',0.8);
c=coneplot(Xq,Yq,Zq,C1,C2,C3,Xq,Yq,Zq,0.5); set(c,'FaceColor','g','EdgeColor','g','FaceAlpha',0.5);
camproj perspective; view(-77,34); box on; light; h = camlight('left');
for i = 1:36; camorbit(10,0); camlight(h,'left'); pause(1); end
function gradientSym = gradient_Sym(V,X,coordinate_system); switch coordinate_system
case {'Cartesian'}
    gradientSym=gradient(V,X);
case {'Cylindrical'}
    gradientSym=[diff(V,X(1)),1/X(1)*diff(V,X(2)),diff(V,X(3))];
```

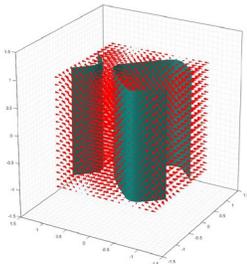
```
case {'Spherical'}
gradientSym=[diff(V,X(1)),1/X(1)*diff(V,X(2)),1/(X(1)*sin(X(2)))*diff(V,X(3))]; end; end
```

Note that the script not only calculates and plots the vector grad (F) but depicts one of the isosurfaces, i.e., the surface where $F = \text{const}$. For the sake of better presentation, this constant was chosen 0.5 while the function F is normalized to $\text{abs}(F) \leq 1$. You can correct the script if you wish. Pay attention and then explain the mutual orientation of the vector fields and isosurfaces.

Now several questions to check the understanding. Please refresh the material in the Appendix, pages 469 - 476.

1. Why the scalar function gradient is a vector, and what does it means?
2. What does the gradient of function indicate?
3. What does it mean if the function gradient is zero at some point?
4. How the gradient link to the function slope?
5. Suppose the orientation of the gradient vector is positive at some point. What does it tell us about the function?
6. What is the orientation of the gradient vector over the isosurface $F = \text{const}$.? Check your answer by looking at the plot.
7. Assuming that the function $F = U$ is the potential U of E-field (see (1.21) in Chapter 1), apply *curl* operator from 3A to calculated $\mathbf{E} = -\text{Grad}_F$ and check that $\text{curl}(\text{grad } U) \equiv 0$ in any of the coordinate systems. What does it tell us about E-fields, their orientation in space, and their ability to move charges (check (A.50) – (A.54))?
8. Assuming again that the function $F = U$ is the potential U of E-fields, apply the *divergence* operator from 2A to calculated $\text{div } \mathbf{E} = \text{div}(\text{grad } U) = \nabla^2 U$ and thereby find the distribution of the E-fields' sources, i.e., the volume density of electric charges (check (A.21) on page 470).

Problem 3. The Divergence of Vector Function. The local function in the Matlab script below is taken from



https://www.mathworks.com/matlabcentral/fileexchange/66259-divergence_sym-v-x-coordinate_system. **Warning!** Copy and paste Matlab script below into a newly created file in **Editor Window**, then save it as DivF.m, for example. The presence of local functions in the script requires Matlab R2016b or later. Push Run button  in Editor Window and proceed with simulation following the instructions on the screen. By definition, the numerical simulation of divergence occurs inside a cube's surface $\max\{|x|,|y|,|z|\}=1$. *Be careful with Matlab syntax entering math expressions!*

Choose the Cartesian coordinate system first and run the example depicted on the screen. The force lines of vector function \mathbf{F} (shown as red cones) cross the cylindrical isosurfaces (dark green) where $\text{div}\mathbf{F} = \text{const}$. For the sake of better presentation, this constant was chosen 0.05 while $\text{abs}(\text{div}\mathbf{F}) \leq 1$. You can adjust the Matlab script if you wish to change it. Pay attention and then explain the mutual orientation of the vector fields and isosurfaces.

Now several questions to check the understanding. Please refresh the material in the Appendix, pages 469-470, and Gauss' Theorem on page 477.

1. Is the divergence of a vector function a vector or scalar? Using the analogy of a water stream, interpret

the concept of "outflowing-ness" and "inflowing-ness."

2. We measured the net flux of 2D E-force lines (red and blue arrows on the left) crossing the cylinder's surface (white dot line). Which of the images corresponds to zero flux?

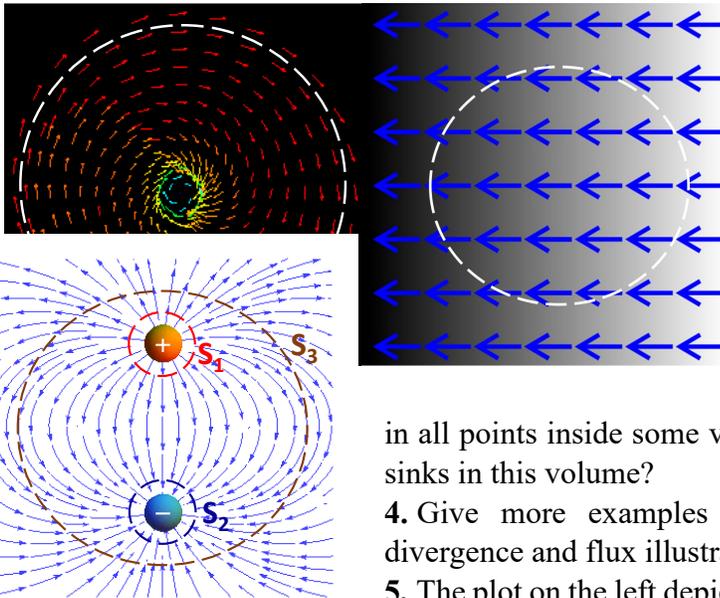
3. What does it mean that the divergence of E-field is positive in all points inside some volume? Can we expect the presence of sources or sinks in this volume? What does it mean that the divergence of E-field is negative

in all points inside some volume? Can we expect the presence of sources or sinks in this volume?

4. Give more examples of E-fields with zero, positive and negative divergence and flux illustrating your answer with E-force lines.

5. The plot on the left depicts 2D E-force lines (blue arrows) starting from the positive charge (red ball) and ending at the negative charge (blue ball). What

could you tell about the sign of divergence of the E-field vector and its flux through the surface of cylinders S_1 , S_2 , and S_3 ?



```
close all; clc; clear; n=input('Enter 1 for Cartesian, 2 for Cylindrical, and 3 for Spherical: ');
```

```
disp('Follow Matlab rules for math expressions. Square brackets [] are required.');
```

```
N=16; [Xq,Yq,Zq]=meshgrid(linspace(-1,1,N)); syms x y z r theta phi
```

```
switch n
```

```
case 1
```

```
F=input(['Enter Vector-fuction of x, y, z (lowercase letters only!) as [Fx, Fy, Fz].!...
```

```
\nFor example, [sin(pi*x), cos(pi*y), 0] ');
```

```
C=divergence_sym(F,[x,y,z],'Cartesian');
```

```
C1=double(subs(C,[x,y,z],[Xq,Yq,Zq])); F1=double(subs(F(1),[x,y,z],[Xq,Yq,Zq]));
```

```
F2=double(subs(F(2),[x,y,z],[Xq,Yq,Zq])); F3=double(subs(F(3),[x,y,z],[Xq,Yq,Zq]));
```

```
case 2
```

```
F=input(['\nEnter vector-fuction of r, phi, z (lowercase letters only!) as [Fr, Fphi, Fz].!...
```

```
\nFor example, 2*r^2*[cos(2*phi) -sin(2*phi) 1] ');
```

```
R=sqrt(Xq.^2+Yq.^2+Zq.^2); Phi=atan2(Yq,Xq); Z=Zq;
```

```
C=divergence_sym(F,[r,phi,z],'Cylindrical'); F0=formula(F);
```

```
C1=double(subs(C,[r,phi,z],[R,Phi,Z])); F1=double(subs(F0(1),[r,phi,z],[R,Phi,Z]));
```

```
F2=double(subs(F0(2),[r,phi,z],[R,Phi,Z])); F3=double(subs(F0(3),[r,phi,z],[R,Phi,Z]));
```

```
case 3
```

```
F=input(['\nEnter vector-fuction of r, theta, phi (lowercase letters only!) as [Fr, Ftheta, Fphi].!...
```

```
\nFor example, [cos(theta)/r sin(theta)/r 0] ');
```

```
R=sqrt(Xq.^2+Yq.^2+Zq.^2); Theta=atan2(sqrt(Xq.^2+Yq.^2),Zq); Phi=atan2(Yq,Xq); Z1=Zq;
```

```
C=divergence_sym(F,[r,theta,phi],'Spherical'); F0=formula(F);
```

```
C1=double(subs(C,[r,theta,phi],[R,Theta,Phi])); F1=double(subs(F0(1),[r,theta,phi],[R,Theta,Phi]));
```

```
F2=double(subs(F0(2),[r,theta,phi],[R,Theta,Phi])); F3=double(subs(F0(3),[r,theta,phi],[R,Theta,Phi]));
```

```
end
```

```
fprintf(2, 'The vector-function is:\n'); F'
```

```
fprintf(2, 'The divergence is:\n'); Div_F=C
```

```
C1=C1/max(abs(C1(:)));
```

```
figure('units','normalized','outerposition',[0 0 1 1]); grid minor, hold on; axis square
```

```
c=coneplot(Xq,Yq,Zq,F1,F2,F3,Xq,Yq,Zq,0.5);
```

```
set(c,'FaceColor','r','EdgeColor','r','FaceAlpha',0.5);
```

```
isosurface(Xq,Yq,Zq,C1,0.05)
```

```
camproj perspective; view(-60,22); box on; light; h = camlight('left');
```

```
for i = 1:36; camorbit(10,0); camlight(h,'left'); pause(1); end
```

```
function Div = divergence_sym(V,X,coordinate_system)
```

```
switch coordinate_system
```

```
case {'Cartesian'}
```

```
Div=divergence(V,X);
```

```

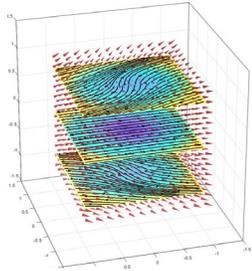
case {'Cylindrical'}
    Div=1/X(1)*diff(X(1)*V(1),X(1))+1/X(1)*diff(V(2),X(2))+diff(V(3),X(3));
case {'Spherical'}
    Div=1/X(1)^2*diff(X(1)^2*V(1),X(1))+1/(X(1)*sin(X(2)))*diff(sin(X(2))*V(2),X(2))+1/(X(1)*sin(X(2)))*diff(V(3),X(3));
end; end

```

Problem 4. The Curl of Vector Function.

The local function in the Matlab script below is taken from https://www.mathworks.com/matlabcentral/fileexchange/66265-curl_sym-v-x-coordinate_system.

Warning! Copy and paste Matlab script below into a newly created file in **Editor Window**, then save it as CurlF.m, for example. The presence of local functions in the script requires Matlab R2016b or later. Push Run button  in Editor Window and proceed with simulation following the instructions on the screen. By definition, the curl's numerical simulation takes place inside a cube's surface $\max\{|x|,|y|,|z|\}=1$. You can adjust the Matlab script if you wish to change it.



Be careful with Matlab syntax entering math expressions! Choose the Cartesian

coordinate system first and run the example depicted on the screen. The normalize profile of $\text{abs}(\mathbf{F}) < 1$ is shown in three cross-section $z = -0.8, 0, 0.8$. The force lines of vector function \mathbf{F} are depicted in red, while the force lines of vector curl \mathbf{F} are black.

```

close all; clc; clear;
n=input('Enter 1 for Cartesian, 2 for Cylindrical, and 3 for Spherical: ');
disp('Follow Matlab rules for math expressions. Square brackets [] are required.')
N=12; [Xq,Yq,Zq]=meshgrid(linspace(-1,1,N)); syms x y z r theta phi
switch n
case 1
    F=input(['Enter Vector-fuction of x, y, z (lowercase letters only!) as [Fx, Fy, Fz].!...
            '\nFor example, [0,0,cos(pi*y/2)*cos(pi*z/2)] '); C=curl_sym(F,[x,y,z],'Cartesian');
    C1=double(subs(C(1),[x,y,z],[Xq,Yq,Zq])); C2=double(subs(C(2),[x,y,z],[Xq,Yq,Zq]));
    C3=double(subs(C(3),[x,y,z],[Xq,Yq,Zq])); F1=double(subs(F(1),[x,y,z],[Xq,Yq,Zq]));
    F2=double(subs(F(2),[x,y,z],[Xq,Yq,Zq])); F3=double(subs(F(3),[x,y,z],[Xq,Yq,Zq]));
case 2
    F=input(['\nEnter vector-fuction of r, phi, z (lowercase letters only!) as [Fr, Fphi, Fz].!...
            '\nFor example, [-r*sin(phi),r*cos(phi),0] ');
    R=sqrt(Xq.^2+Yq.^2+Zq.^2); Phi=atan2(Yq,Xq); Z=Zq; C=curl_sym(F,[r,phi,z],'Cylindrical');
    C1=double(subs(C(1),[r,phi,z],[R,Phi,Z])); C2=double(subs(C(2),[r,phi,z],[R,Phi,Z]));
    C3=double(subs(C(3),[r,phi,z],[R,Phi,Z])); F1=double(subs(F(1),[r,phi,z],[R,Phi,Z]));
    F2=double(subs(F(2),[r,phi,z],[R,Phi,Z])); F3=double(subs(F(3),[r,phi,z],[R,Phi,Z]));
case 3
    F=input(['\nEnter vector-fuction of r, theta, phi (lowercase letters only!) as [Fr, Ftheta, Fphi].!...
            '\nFor example, [2*r*cos(theta),-3*r*sin(theta),r*cos(phi)] ');
    R=sqrt(Xq.^2+Yq.^2+Zq.^2); Phi=atan2(Yq,Xq); Theta=atan2(sqrt(Xq.^2+Yq.^2),Zq);
    C=curl_sym(F,[r,theta,phi],'Spherical'); C0=formula(C); C1=double(subs(C0(1),[r,theta,phi],[R,Theta,Phi]));
    C2=double(subs(C0(2),[r,theta,phi],[R,Theta,Phi])); C3=double(subs(C0(3),[r,theta,phi],[R,Theta,Phi]));
    F1=double(subs(F(1),[r,theta,phi],[R,Theta,Phi])); F2=double(subs(F(2),[r,theta,phi],[R,Theta,Phi]));
    F3=double(subs(F(3),[r,theta,phi],[R,Theta,Phi])); end
disp('You entered function F'); F'
FF=sqrt(F1.^2+F2.^2+F3.^2); FF=FF/max(abs(FF(:))); Curl_F=C'
figure('units','normalized','outerposition',[0 0 1 1]); grid on, hold on; axis square
if n==2; quiver3(Xq,Yq,Zq,C1,C2,C3,1,'LineWidth',2,'MaxHeadSize',1,'color','g'); end
hs1=slice(Xq,Yq,Zq,FF,[],[],[-0.8,0,0.8]); set(hs1,'FaceColor','interp','EdgeColor','none');
if n~=2; hs2=streamslice(Xq,Yq,Zq,C1,C2,C3,[],[],[-0.8,0,0.8],1); set(hs2,'Color','k','LineWidth',2); end
c=coneplot(Xq,Yq,Zq,F1,F2,F3,Xq,Yq,Zq,0.5); set(c,'EdgeColor','none','FaceColor','r');
camproj perspective; view(-106,26); box on; light; h = camlight('left');
for i = 1:36; camorbit(10,0); camlight(h,'left'); pause(1); end
function CurlSym = curl_sym(V,X,coordinate_system)
switch coordinate_system
case {'Cartesian'}
    CurlSym=curl(V,X);
case {'Cylindrical'}
    CurlSym=[1/X(1)*diff(V(3),X(2))-diff(V(2),X(3)),...
            diff(V(1),X(3))-diff(V(3),X(1)),...
            1/X(1)*diff(X(1)*V(2),X(1))-1/X(1)*diff(V(1),X(2))];
case {'Spherical'}

```

```

CurlSym=[1/(X(1)*sin(X(2)))*(diff(V(3)*sin(X(2)),X(2))-diff(V(2),X(3))),...
1/X(1)*(1/sin(X(2))*diff(V(1),X(3))-diff(X(1)*V(3),X(1))),...
1/X(1)*(diff(X(1)*V(2),X(1))-diff(V(1),X(2)));

```

```
end; end
```

Now several questions to check the understanding. Please refresh the material in the Appendix, pages 471, 475 - 477.

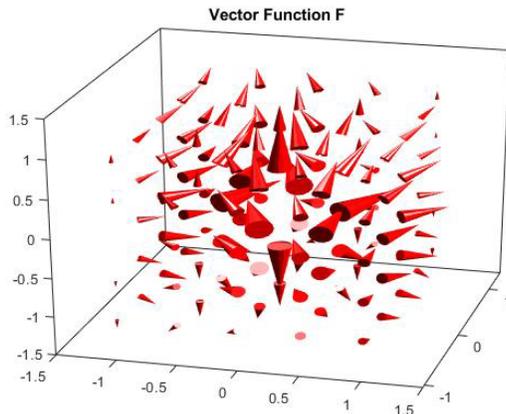
Problem 5. Electric Potential of Conservative and Non – Conservative E-Fields.

Please refresh the material of Sections 1.6.1 – 1.6.3 in Chapter 1, Section 2.2.8 in Chapter 2, and the comments on pages 475 – 476 in Appendix.

Warning! Copy and paste Matlab script below into a newly created file in **Editor Window**, then save it as Potential.m, for example. The presence of local functions in the script requires

Matlab R2016b or later. Push Run button  in Editor Window and proceed with simulation following the instructions on the screen. *Be careful with Matlab syntax entering math expressions!* The script animates the effect of endpoint 2 (check expression (1.21) of Chapter 1) and the path taken between points 1 and 2 on the work done by E-field down the path. For the sake of simplicity, starting point 1 for both

ways is fixed while point 2 (red or blue) moves along the designated curve, helical segment (red), or vertical line (blue), respectively. The endpoint 2 of both curves is shared.



```

close all; clc; clear; syms x y z t real; R0=sqrt(x^2+y^2+z^2);
Rp=sqrt((x+0.25)^2+(y+0.25)^2+(z+0.25)^2); Rm=sqrt((x-0.25)^2+(y-0.25)^2+(z-0.25)^2);
fprintf(2,['Input below Vector-function E(x,y,z) (lowercase letters for x,y,z) as [Ex, Ey, Ez].'\n\n
You can type R0=sqrt(x^2+y^2+z^2); Rp=sqrt((x+0.25)^2+(y+0.25)^2+(z+0.25)^2);\n\n
Rm=sqrt((x-0.25)^2+(y-0.25)^2+(z-0.25)^2); \n as variables. For example,\n\n
[x-0.25,y-0.25,z-0.25]/Rm^3-[x+0.25,y+0.25,z+0.25]/Rp^3' ...
\n or [x^2-y^2,abs(y-x),z]/R0^2 \n\n'); E=input('Enter Vector-function here E = ');
figure(1); vectorfield3(E,-1:.5:1,-1:.5:1,-1:.5:1); camproj perspective; box on; light; camlight('left');
title('\bfE-force Line Durection and Magnitude'); view(15,25);
Curl=curl(E,[x y z]); CurlMag=simplify((Curl(1)^2+Curl(2)^2+Curl(3)^2));
cond = subs(CurlMag,{x,y,z},{1,1,1}); if cond==0; fprintf(2,'E-Field is Conservative\n'); else
fprintf(2,'E-Field is Non-Concervative\n'); end; H=[cos(t),sin(t),t]; H_1=[1,0,t];
Fpar=subs(E,[x,y,z],H); Fpar_1=subs(E,[x,y,z],H_1);
F_dl=simplify(dot(Fpar,diff(H,t))); F_dl_1=simplify(dot(Fpar_1,diff(H_1,t))); ti=0:pi/20:4*pi;
Curve=double(subs(H,t,ti)); Curve_1=double(subs(H_1,t,ti)); L=length(ti);
xi=Curve(1:L); yi=Curve(L+1:2*L); zi=Curve(2*L+1:end);
xi1=Curve_1(1:L); yi1=Curve_1(L+1:2*L); zi1=Curve_1(2*L+1:end);
for i=1:length(ti); integral(i)=quadl(inline(vectorize(F_dl)),1e-3,ti(i));
integral_1(i)=quadl(inline(vectorize(F_dl_1)),1e-3,ti(i)); end
f2=figure; movegui(f2,'west'); hold all;
camproj perspective; box on; light; h = camlight('left');
plot3(xi,yi,zi,'r',xi1,yi1,zi1,'b'); view(3); title('\bfPath along Helix and Line');
text(xi(1),yi(1),zi(1),'\bf Point 1'); text(xi(end),yi(end),zi(end),'\bf Point 2');
for i=1:length(ti); h=scatter3(xi(i),yi(i),zi(i),'filled','r'); h.SizeData = 50;
h1=scatter3(xi1(i),yi1(i),zi1(i),'filled','b'); h1.SizeData = 50; drawnow; view(-60,22); pause(0.1)
end
legend('\bfHelical Curve','\bfVertical Path','Location','Best'); hold off;
f3=figure; movegui(f3,'east'); hold all; grid minor
plot(ti,integral,'r',ti,integral_1,'b')
title('\bfRelative Electric Potential/Work along Helix and Line');
text(ti(end),integral(end),'\bf Point 2'); text(ti(end),integral_1(end),'\bf Point 2');
for i=1:length(ti); h=scatter3(ti(i),integral(i),'filled','r'); h.SizeData = 50;
h1=scatter3(ti(i),integral_1(i),'filled','b'); h1.SizeData = 50;
drawnow; pause(0.1); end
legend('\bfHelical Curve','\bfVertical Path','Location','Best'); hold off
function vectorfield3(F,xval,yval,zval); [xg,yg,zg] = meshgrid(xval,yval,zval);

```

```
F1f = inline(vectorize(F(1)),'x','y','z'); F2f = inline(vectorize(F(2)),'x','y','z');
F3f = inline(vectorize(F(3)),'x','y','z'); F1g = F1f(xg,yg,zg); F2g = F2f(xg,yg,zg); F3g = F3f(xg,yg,zg);
c=coneplot(xg,yg,zg,F1g,F2g,F3g,xg,yg,zg,1); set(c,'EdgeColor','none','FaceColor','r');
camproj perspective; box on; light; camlight('left'); end
```

Run one of the E-field vector example depicted on the screen. The first example corresponds to the E-field of two point charges, while the second one was chosen as a fiction non-conservative field. The central Figure 1 illustrates the E-force lines of vector function \mathbf{E} you have entered. The cone sizes are proportional to E-field intensity. The appeared on display the red message enlightens the status of this field. Figure 3 illustrates how the potential U_e of E-field varies along the curves. The movement of red and blue markers in Figures 2 and 3 is synchronized.

Problem 6. Static Electric Fields

6.1. What is the strength and direction of the static and uniform electric field \mathbf{E} [V/m] (use expression (1.17)) pushing an electron of $1.60217662 \times 10^{-19}$ [C] in the positive direction of the z -axis by the force 1×10^{-15} [N]? Compare this field strength with the air breakdown strength 3×10^6 [V/m]. Might we watch a light spike in air or corona as multiple electrons are accelerated simultaneously? *Hint.* Search Wikipedia.

6.2. Using data from problem 1.1E and Newton's Second Law of motion $\mathbf{F} = m\mathbf{a}$, find the electron acceleration of $m = 9.10938356 \times 10^{-31}$ [kg], its velocity, and traveled distance for a time period of 10^{-8} [s]. Compare this velocity with speed of light $c = 299\,792\,458$ [m/s]. Is it possible for such an accelerating electron to reach the speed of light? If your answer is yes, how long does it take?

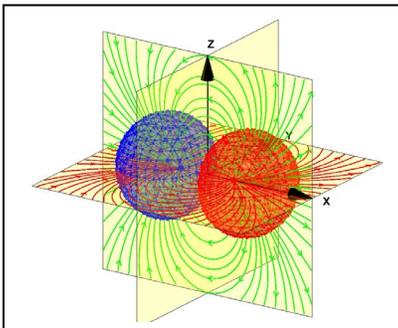
6.3. Using data from problem 1.1E, find the energy W_e [J] (use the expression (1.19) in Chapter 1) that the electric field transfers into the kinetic energy of accelerating an electron. Calculate the kinetic energy of the electron as $W_k = mv^2/2$ and check that $W_k = W_e$. Recalculate W_e in the electron-volt [eV] assuming that 1 [eV] = 1.60×10^{-19} [J] and explain the meaning of electron-volt units.

6.4. Using data from the problem 1.1E - 1.3E, find the electric potential U_e [V] (use expression (1.21) in Chapter 1) that accelerated the electron.

6.5. Copy, paste, and execute the following script into Matlab Command Window to check your calculations.

```
clc; close all; clear; e = 1.60217662e-19; m = 9.10938356e-31; c = 299792458; F=1e-13; t=1e-9;
E=F/e;a=F/m;v=a*t;ratio=v/c;dist=a*t^2/2;We=e*E*dist;Wk=m*v^2/2;WeV=We/1.6e-19;Ue=We/e;
X1 = ['|E| = ',num2str(E),' V/m;',' a = ',num2str(a),' m/s^2;',' v = ',num2str(v),' m/s;'];disp(X1);
X2 = ['v/c = ',num2str(ratio),';',' dist = ',num2str(dist),' m;',' We = ',num2str(We),' J;'];disp(X2);
X3 = ['Wk = ',num2str(Wk),' J;',' WeV = ',num2str(WeV),' eV;',' Ue = ',num2str(Ue),' V;'];disp(X3);
```

6.6. Sketch out the equipotential lines of electric potential U_e and lines of E-force produced by static point charges located inside the cube $|x| \leq 2$ [m], $|y| \leq 2$, $|z| \leq 2$ [m]. For example, **a)** single charge in origin $Q=[1]$, $x=[0]$, $y=[0]$, $z=[0]$, **b)** dipole in XY-plane $Q=[1,-1]$, $x=[0,0]$, $y=[1,-1]$, $z=[0,0]$, and **c)** quadrupole in XY-plane $Q=[1,-1,1,-1]$, $x=[-1,1,1,-1]$, $y=[1,1,-1,-1]$, $z=[0,0,0,0]$. Copy, paste, and execute the following script (see next page) into Matlab Command Window. Compare the plots with your sketches and clarify the differences if you found them.



6.7 If you like to enjoy 3D animation of multiple charges' static fields, go to <https://1drv.ms/f/s!AjtsKS-uvNP1avsbUFY9UsxLVVE>, click subsequently on the icon Document and MultiCharge. Mark, copy and download MultiCharges.m file to a newly created directory named MultiCharge, for

example. Then open the MultiCharges.m file in Matlab and run the animation. The screenshot above illustrates it. Now it is time to answer the questions.

1. Explain how to estimate E-field's relative strength and direction (see Section 1.6.3 in Chapter 1) using this plot.
2. Show that the E-field vector from point charges can be obtained by taking the vector sum of the individual charges' E-fields.
3. Find the total flux of E-fields flowing through the surface of the sphere surrounding each of the charges only and their different combinations (see Sections 1.6.4 and 1.6.8 in Chapter 1).
4. Ask the instructor for additional questions.

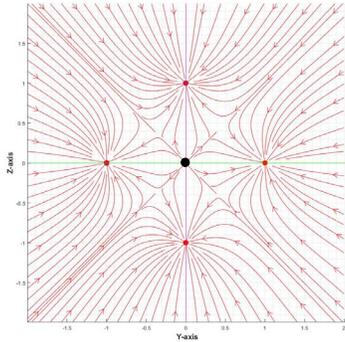
```

clc; close all; clear
fprintf(2,'All charges must be put inside volume -2<=X<=2,-2<=Y<=2,-2<=Z<=2 [m]\n')
fprintf(2,'\n Total number of charges is equal M \n ')
fprintf(2,'\n For example, for dipole in XY-plane M=2 and Vector Q is [1,-0.5]\n')
fprintf(2,'\n Vector X is [0.5 0.5], vector Y is [0.5 -0.2], and vector Z is [0 0] \n')
fprintf(2,'\n Square brackets and Commas are mandatory\n')
fprintf(2,'\n Hint. Before entering data draw all charges and write their coordinate-vectors on paper in required format\n\n')
Q=input('Enter all charges Q in [C] as a vector [Q(1),Q(2),Q(3),...,Q(M)]= ');
xx=input('Enter all X-coordinates in [m] as a row-vector [x(1),x(2),x(3),...,x(M)]= ');
yy=input('Enter all Y-coordinates in [m] as a row-vector [y(1),y(2),y(3),...,y(M)]= ');
zz=input('Enter all Z-coordinates in [m] as a row-vector [z(1),z(2),z(3),...,z(M)]= ');
k=8.987E9; N=150; x=linspace(-1,1,N)*2; y=linspace(-1,1,N)*2; z=linspace(-1,1,N)*2;
[X,Y,Z]=meshgrid(x,y,z); Ue=zeros(size(X)); for ii=1:numel(Q)
hy=sqrt((xx(ii)-X).^2+(yy(ii)-Y).^2+(zz(ii)-Z).^2); Ue=Ue+Q(ii)/hy; end
Ue=k*Ue; V=log(abs(Ue)); [Ex,Ey,Ez]=gradient(Ue,2/N); E=sqrt(Ex.^2+Ey.^2+Ez.^2); LE=log(E);
figure('units','normalized','outerposition',[0 0 1 1]); grid minor; hold on;
h3 = streamslice(X,Y,Z,Ex,Ey,Ez,0,[],,2);set(h3,'Color','r'); h4 = streamslice(X,Y,Z,Ex,Ey,Ez,[],0,[],2);set(h4,'Color','m')
h5 = streamslice(X,Y,Z,Ex,Ey,Ez,[],,0,2);set(h5,'Color','g'); for jj=1:numel(Q); if Q(jj)>0;
plot3(xx(jj),yy(jj),zz(jj),'o','MarkerSize',10,'MarkerFaceColor','r'); else
plot3(xx(jj),yy(jj),zz(jj),'o','MarkerSize',10,'MarkerFaceColor','b'); end; end
xlabel('X-axis','FontWeight','bold','FontSize',14); ylabel('Y-axis','FontWeight','bold','FontSize',14);
zlabel('Z-axis','FontWeight','bold','FontSize',14); axis square; view(-74,16);
title('\bfe-force Lines in XY-, XZ- and YZ-plane');
figure('units','normalized','outerposition',[.2 .2 .6 .6]); grid minor; hold on;
colormap(jet(64)); hh=colorbar; ylabel(hh,'\n(|Ue(x,y,z)| [V/m])','FontWeight','Bold','FontSize',15);
h1 = contourslice(X,Y,Z,V,[],,0,100); h2 = contourslice(-X,Y,Z,V,0,[],,100);
xlabel('X-axis','FontWeight','bold','FontSize',14); ylabel('Y-axis','FontWeight','bold','FontSize',14);
zlabel('Z-axis','FontWeight','bold','FontSize',14); axis square; view(-69,27);
title('\bfequipotential Lines of Potential Ue in XZ- and YZ-plane';Logarithmic scale});
vert = [-2 0 -2;2 0 -2;2 0 -2;-2 0 -2;-2 0 2;2 0 2;2 0 2;-2 0 2];
fac = [1 2 6 5;1 2 6 5;1 2 6 5;1 2 6 5;1 2 6 5;1 2 6 5;1 2 6 5];
ph=patch('Vertices',vert,'Faces',fac,'FaceVertexCData',hsv(6),'FaceColor','flat');alpha(ph,0.1)
vert = [0 -2 -2;0 2 -2;0 2 -2;0 -2 -2;0 -2 2;0 2 2;0 2 2;0 -2 2];
fac = [1 2 6 5;1 2 6 5;1 2 6 5;1 2 6 5;1 2 6 5;1 2 6 5];
ph1=patch('Vertices',vert,'Faces',fac,'FaceVertexCData',hsv(6),'FaceColor','flat');alpha(ph1,0.1)
v1=[0,0,2]; v2=[0,0,-2]; v=[v1;v2]; plot3(v(:,1),v(:,2),v(:,3),'-k','LineWidth',2)
figure('units','normalized','outerposition',[.3 .3 .6 .6]); grid minor; hold on;
colormap(jet(64)); hh=colorbar; ylabel(hh,'\n(|E(x,y,z)| [V/m])','FontWeight','Bold','FontSize',15);
contourslice(X,Y,Z,LE,[],,zz,80); contourslice(-X,Y,Z,LE,0,[],,80);
xlabel('X-axis','FontWeight','bold','FontSize',14); ylabel('Y-axis','FontWeight','bold','FontSize',14);
zlabel('Z-axis','FontWeight','bold','FontSize',14); axis square; view(-21,19);
title('\bfequipotential Lines of E-field Logarithm');

```

6.8. Put URL <http://www.falstad.com/vector3de/> in your browser and press Enter. Click "Full-screen version" on the bottom of the opened page. In the "Field selection" on the right top corner, choose "dipole" and "Display: Field Lines or Equipotentials." Go to "Show Z Slice" and compare with the plots you have received solving problem 1.5E. Examine different combinations of charges and material bodies. Explain images. Play and enjoy.

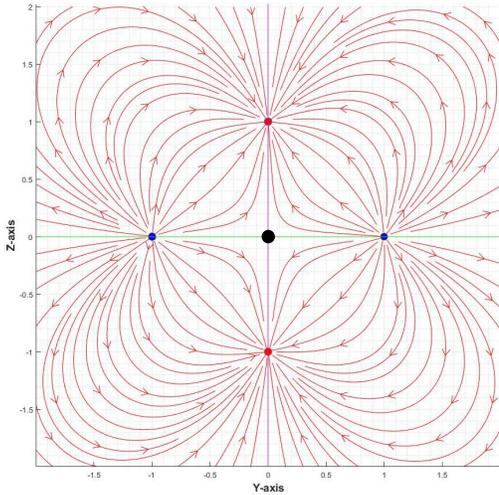
6.9. The Matlab script from Problem 6 was run into the Matlab Command Window. The figure depicts the



obtained image of E-force lines initiated by the quadrupole, i.e., 4 charges are arranged on the corners of a square. The black spot in origin is a motionless field sensor #1 (see Section 1.4.1 in Chapter 1). After analyzing the drawing, answer the questions:

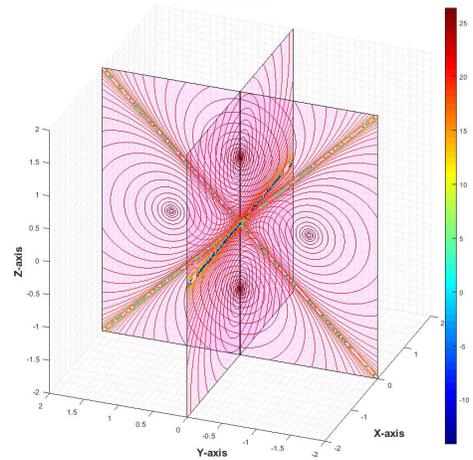
1. What is the charges' polarity? Which one is positive and negative?
2. What could you tell about the charges' values?
3. What force might move this sensor and where?
4. How much is the E-field flux through the closed surface that includes all charges?
5. How much is the E-field flux through the closed surface that includes the sensor only?

6.10. The script from problem 1.6E was rerun into Matlab Command Window. The figure on the left depicts



the obtained image of the E-force lines initiated by the quadrupole.

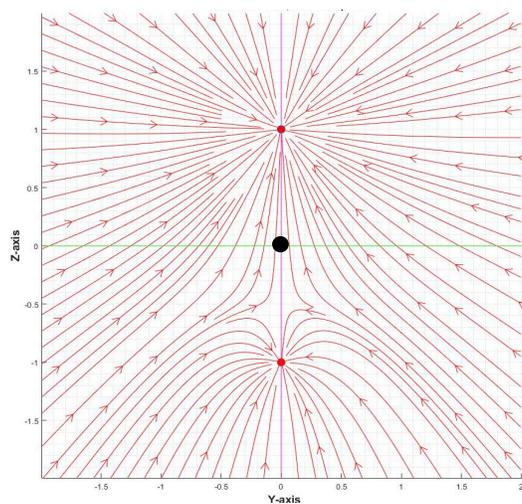
Analyzing the drawing, answer the questions from problem 1.8E. Then look at the picture on your right that illustrates the equipotential lines of electric potential (really the natural



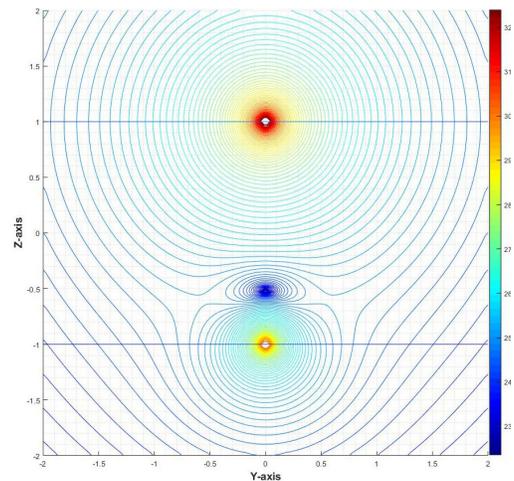
logarithm to get the more revealing image) for the same

quadrupole. What is the potential value along the two diagonals in the YZ-plane and along the line $Z = 0$ in the XZ-plane? Suppose the infinitely thin sheets of copper are put along the diagonals in parallel to the y -axis. Does it change the E-field structure, and how? (*Hint.* Check boundary conditions for the E-field tangential components in Table 2.2 of Chapter 2). Give detailed explanations and supporting the drawing of E-force lines.

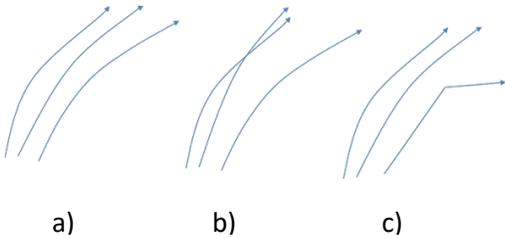
6.11. The script from problem 1.6E was rerun into Matlab Command Window. The figure on the left depicts the obtained image of the E-force lines initiated by the two charges. Analyzing the drawing, answer the



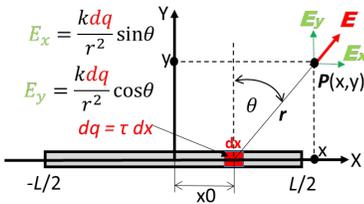
questions from problem 1.8E. Then look at the picture on your right that illustrates the equipotential lines of E-fields (really the natural logarithm to get the more revealing image). What could you tell



about the charges' value? How to explain the deep drop in E-field intensity (blue spot) and its location shifted to one of the charges? Which one of the images shown below of E-force lines reflects correctly a physically sound E-field structure? (*Hint.* Read Section 1.6.3 and check the E-field definition (1.18) in Chapter 1)? Give reasons supporting your answer.



Problem 7. E-near-fields of Electrically Short Non-conducting Rod. Suppose a charge Q spreads uniformly along the x -axis with linear density $\tau = Q/L$, where L is the charge line's total length². E-field due to the total charge Q can be calculated by superposing (really integrating) the E-field of infinitesimal charge dq using the equation (1.24) in Chapter 1. The trigonometric functions are equal $\sin \theta = (x - x_0)/r$ and $\cos \theta = y/r$ where $r^2 = (x - x_0)^2 + y^2$. Allowing x_0 to vary from $-L/2$ to $L/2$, write down the integral for each E-field component. Sketch out the 2D equipotential lines of E-force produced by the charged line. Then copy, paste, and run the



following script into Matlab Command Window and compare the plot with your sketch. Explain differences if you found them.

```

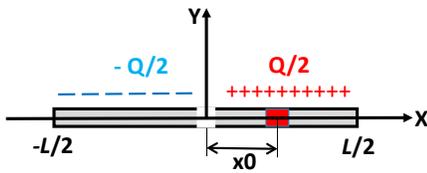
clc; close all; clear
k = 8.987E9; display(' Length L of finite line of charge L<=9m')
L=input('Enter the length L of finite line of charge in [m]: ');
tau=input('Enter the charge density of charges in [C/m]: ');
[X,Y] = meshgrid(-2*L:0.020011:2*L); x0=linspace(-L/2,L/2,101);
for jj=1:size(x0,2); r=sqrt((x0(jj)-X).^2+Y.^2); dV(jj,:)=k*tau./r; end
Ue=squeeze(trapz(dV,1)); [Ex,Ey]=gradient(Ue);
F1=figure('units','normalized','outerposition',[0 0 1 1]); grid minor; hold all
hLines = streamslice(X,Y,Ex,Ey,5); set(hLines,'Color','r'); xlabel('-5 \leq X \leq 5','FontWeight','bold');
ylabel('-5 \leq Y \leq 5','FontWeight','bold'); axis tight
plot(x0,0,'ko','MarkerSize',20,'MarkerEdgeColor','k','MarkerFaceColor','k')
annotation(F1,'textarrow',[0.42 0.48],[0.42 0.50],'TextLineWidth',2,'FontSize',20,'FontWeight','Bold',...
'String',{'Charged Line'},'HeadStyle','deltoid','LineWidth',2,'Color','k');
contour(X,Y,Ue,40); hh=colorbar('fontsize',12);
ylabel(hh,'Potential Intensity Ue(X,Y)','FontWeight','Bold','FontSize',15);
xlabel('X-axis [m]','FontSize',15,'FontWeight','bold'); ylabel('Y-axis [m]','FontSize',15,'FontWeight','bold');
axis square; hold off; figure(2); h2=mesh(X,Y,Ue); g2=colorbar('fontsize',12);
xlabel(g2,'Electric Potential Ue [V]','FontWeight','Bold','FontSize',15);
xlabel('x-axis','fontsize',14); ylabel('y-axis','fontsize',14); zlabel('Electric Potential [V]','fontsize',14)
h3=title('Electric Potential Distribution, V(x,y) [V]'); set(h3,'fontsize',14); view(121,24); grid minor

```

1. Examine two individual cases of the short line ($L = 0.1\text{m}$, for example) and a relatively long line of $L = 9\text{m}$. Compare plots and explain the differences in E-field structures.
2. What kind of E-field structure could you expect in the case of an infinitely long line as $L \rightarrow \infty$?
3. Examine the case of a negative charge.
4. Find the total flux of E- and D-fields flowing through the surface of the sphere surrounding the line of charges.
5. If you like a challenge, present in closed form all integrals numerically calculated in the script. Ask the instructor for additional questions.

² Bear in mind that the charges of the same sign “hate” each other by pushing them away that makes practically impossible to keep such charges in static position at the same area. That is why all the following exercises with static or defined upfront charge distribution are just intended to demonstrate and clear up some physical principles and generally serve as rough approximation of real-world problems.

Problem 8. E-near-fields of Electrically Short Dipole. Assume that the charge line comprises two separate halves carrying equal but opposite charges $\pm Q/2$. Sketch out the 2D equipotential lines of E-force produced by these charged lines. Then copy, paste, and run the script below into Matlab Command Window. Explain differences if you found them.



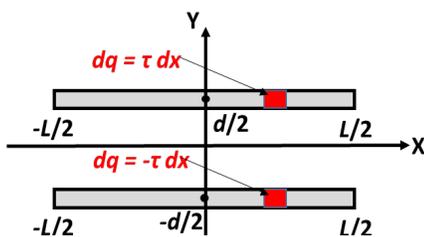
```

clc; close all; clear; k = 8.987E9; a=5;
display(' Length L of finite line of charge L<=9m'); L=input('Enter the length L of finite line of charge in [m]: ');
tau=input('Enter the charge linear density of charges in [C/m]: ');
[X,Y] = meshgrid(-L:0.02:L); x0=linspace(-L/2,L/2,200);
for jj=1:size(x0,2); r(jj,:)=sqrt((x0(jj)-X).^2+Y.^2); x1(jj,:)=X-x0(jj);
    if x0(jj)>=0; dEx(jj,:)=k*tau*x1(jj,:)./r(jj,:).^3; dEy(jj,:)=k*tau./r(jj,:).^3;
    else; dEx(jj,:)=k*tau*x1(jj,:)./r(jj,:).^3; dEy(jj,:)=k*tau./r(jj,:).^3; end; end
Ex=squeeze(trapz(dEx,1)); Ey=Y.*squeeze(trapz(dEy,1));
F1=figure('units','normalized','outerposition',[0 0 1 1]); hax=axes; grid minor; hold all
hLines = streamslice(X,Y,Ex,Ey,5); set(hLines,'Color','r'); xlabel('-5 \leq X \leq 5','FontWeight','bold');
ylabel('-5 \leq Y \leq 5','FontWeight','bold'); plot(x0,0*x0,'k*'); axis tight
annotation(F1,'textarrow',[0.42 0.48],[0.42 0.51],'TextLineWidth',2,'FontSize',12,...
'String',{ 'Charged Line' },'HeadStyle','deltoid','LineWidth',2,'Color','k');

```

1. Describe how to estimate E-field's relative strength and direction (see Section 1.6.3 in Chapter 1) using this plot. Mark area with the highest intensity of E-field.
2. Find the total flux of E- and D-fields flowing through the surface of the sphere surrounding the line of charges. Explain the results and compare them with the near-field E-field structure around an elementary electric dipole depicted in Figure 4.3.1 of Chapter 4.
3. If you like a challenge, present all integrals in closed form. Then demonstrate analytically that the dominant components of dipole E-near-fields (see (5.20) and (5.21) in Chapter 5) and this charged line is almost identical, and both deviate as $\sim 1/r^2$ and $\sim 1/r^3$ nearby.
4. Show that the static and indeed non-radiating charged line is not able to create the E-far-field component $\sim 1/r$.
5. Ask the instructor for additional questions.

Problem 9. E-fields of Electrically Short Two-wire Line. Suppose a charge $+Q$ spreads uniformly along the x -axis with linear density $\tau = Q/L$, and a charge $-Q$ extends uniformly along the same x -axis with linear density $\tau = -Q/L$, as shown here. $L \leq 1.5\text{m}$ is the total length of each of the charge lines, and $d = L/8 \ll L$ (you can change it in Matlab script) is the separation between them. Sketch out the 2D equipotential lines of E-force produced by the charged lines. Then copy, paste, and run into Matlab Command Window; the script slightly modified from 1.12E to consider the second charge line. Explain differences if you found



them.

```

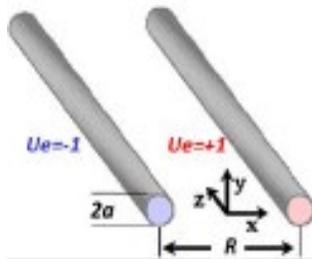
clc; close all; clear
display(' Length L of finite line of charge L<=1.5m'); L=input('Enter the length L [m] of finite line of charge: ');
d=input('Enter the separation d < L [m]: '); tau=input('Enter the linear density of charges [C/m]: ');
x=linspace(-L,L,100); y=linspace(-2*d,2*d,100); k = 8.987E9; [X,Y] = meshgrid(x,y); x0=linspace(-L/2,L/2,100);
for jj=1:size(x0,2)
    r1(jj,:)=sqrt((x0(jj)-X).^2+(Y-d/2).^2); xx(jj,:)=X-x0(jj);
    r2(jj,:)=sqrt((x0(jj)-X).^2+(Y+d/2).^2);
    dE1x(jj,:)=k*tau*xx(jj,:)./r1(jj,:).^3; dE1y(jj,:)=k*tau./r1(jj,:).^3;
    dE2x(jj,:)=k*tau*xx(jj,:)./r2(jj,:).^3; dE2y(jj,:)=k*tau./r2(jj,:).^3; end
dEx=dE1x+dE2x; Ex=squeeze(trapz(x,dEx,1)); Ey=(Y-d/2).*squeeze(trapz(y,dE1y,1))+(Y+d/2).*squeeze(trapz(y,dE2y,1));
figure('units','normalized','outerposition',[0 0 1 1]); hax=axes;grid minor; hold all;
hLines = streamslice(X,Y,Ex,Ey,10);set(hLines,'Color','r');

```

```
text(0,d/2+0.1,'+Q ','Color','red','FontSize',50); text(0,-d/2-0.1,'-Q ','Color','blue','FontSize',50)
xlabel('-1 \leq X \leq 1','FontWeight','bold'); ylabel('-1 \leq Y \leq 1','FontWeight','bold');
plot(x0,d/2,'k*');plot(x0,-d/2,'k*'); axis tight
```

1. Examine the E-field structure in the gap between the charge lines. Explain why it looks like an E-field distribution (Section 6.1.1 in Chapter 6) along the two-wire line length.
2. Extrapolate this structure to E-field's image between the capacitor plates shown in Figure 3.1.10b in Chapter 3.
3. Draw several equipotential lines of potential.
3. Find the total flux of E- and D-fields flowing through the surface of the sphere surrounding the lines of charges.
4. Ask the instructor for additional questions.

Problem 10. E-field around Two Parallel Conductive Wires of Circular Cross-sections. Suppose two



straight conductive circular wires of identical diameter a and infinite length are surrounded by air. Their axes are separated by a distance R , parallel to the z -axis and symmetrically located relative to the z - and x -axis. The battery (not shown) supports an equal and opposite potential of $1[V]$ on each wire. The following Matlab script calculates and pictures the potential and E-force lines in space outside wires. Sketch out the 2D equipotential lines of potential and E-force lines produced by these wires in the XY -plane. Then copy, paste, and run into Matlab Command Window. Explain differences if you found them.

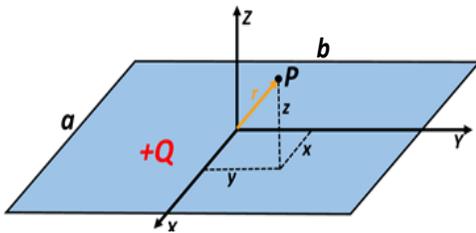
```
clc; close all; clear; % http://bourabai.kz/assis/Revista-Bras-Ens-Fis-V21-p469-475(1999).pdf
display('Two Parallel Conductive Wires of Circular Cross-sections and Equal Radius')
a=input('Enter the radius a of wires [m]: ');
R=input('Enter the separation R > 2a (!) of wires [m]: ');
NN=400; x=linspace(-2*R,2*R,NN);y=linspace(-2*R,2*R,NN);
[X,Y] = meshgrid(x,y); Coeff=1/(2*log((R-sqrt(R^2-4*a^2))/(2*a)));
c1=(X-sqrt(R^2-4*a^2)/2).^2+Y.^2; c2=(X+sqrt(R^2-4*a^2)/2).^2+Y.^2;
Ue=Coeff*log(c1./c2);[Ex,Ey]=gradient(Ue,4*R/NN,4*R/NN); Umax=max(max(Ue));
Ue((X-R/2).^2+Y.^2<=a^2 | (X+R/2).^2+Y.^2<=a^2) = nan; C=8.854e-12*pi/acosh(R/(2*a));
F1=figure('units','normalized','outerposition',[0 0 1 1]); grid minor; hold all
hLines = streamslice(X,Y,-Ex,-Ey,4); set(hLines,'Color','r');
xlabel('X-axis [m]','FontWeight','bold'); ylabel('Y-axis [m]','FontWeight','bold');
th=0:pi/50:2*pi; x1=a*cos(th)+R/2; y1=a*sin(th); fill(x1,y1,'k');
x1=a*cos(th)-R/2+.005; y1=a*sin(th); fill(x1,y1,'k'); axis square
t1=text(-0.5*a,-1.2*a,'Wires'); set(t1,'FontSize',20,'FontWeight','Bold')
contour(X,Y,Ue,80); h1=colorbar('southoutside','fontsize',12); plot(0,-2*R:0.01:2*R,'k','LineWidth',1);
xlabel(h1,'Electric potential Ue(x,y) [V]','FontWeight','Bold','FontSize',15);
t2=text(-R/2-a,1.8*a,'Ue = -1V'); set(t2,'FontSize',20,'FontWeight','Bold','Color','b')
t3=text(R/2-a,1.8*a,'Ue = +1V'); set(t3,'FontSize',20,'FontWeight','Bold','Color','r')
xlabel('X-axis [m]','FontSize',15,'FontWeight','bold'); ylabel('Y-axis [m]','FontSize',15,'FontWeight','bold');
title(['Line Capacitance per meter = ',num2str(C),' [F/m]','Color','k','FontSize',20)
hold off; figure(2); h2=meshc(X,Y,Ue); g2=colorbar('fontsize',12); zlim([-Umax Umax]); axis tight
xlabel(g2,'Electric Potential Ue [V]','FontWeight','Bold','FontSize',15);
xlabel('x-axis','fontsize',14); ylabel('y-axis','fontsize',14); zlabel('Electric Potential [V]','fontsize',14)
h3=title('Electric Potential Distribution Ue(x,y) [V]'); set(h3,'fontsize',14); view(121,24); grid minor
vert = [0 -2*R -1;0 2*R -1;0 2*R -1;0 -2*R -1; 0 -2*R 1;0 2*R 1;0 2*R 1;0 -2*R 1];
fac = [1 2 6 5;1 2 6 5;1 2 6 5;1 2 6 5;1 2 6 5;1 2 6 5];
ph=patch('Vertices',vert,'Faces',fac,'FaceVertexCData',hsv(6),'FaceColor','flat');alpha(ph,0.2); view(-168,15)
```

1. Examine the E-field structure. Explain why it is identical to shown in Figure 6.1.1 of Chapter 6.
2. Please read Section 6.1.1 about TEM that explains this phenomenon. Describe how to estimate E-field's relative strength and direction (see Section 1.6.3 in Chapter 1) using this plot. Mark area with the highest intensity of E-field.
3. Could we expect and where Corona or electrical air breakdown as soon as the wire potential, as well as E-fields, become too high?
4. Knowing that E-field's normal component on the metal surface is equal to the density of electrical charges (read Section 2.3.2 in Chapter 2), sketch the qualitative profile of these charges on the wire surfaces.

5. Explain why no electric field may exist inside a conductor (read Section 2.2.7 in Chapter 2).
6. Check on the graph that E-field's tangential component is zero there (see Figure 2.3.5 in Chapter 2).
7. Find the total flux of E- and D-fields flowing through the cylinder's surface surrounding the two-wire line.
8. Evaluate the potential value along the symmetry line (black dotted) in the first figure and the second's pink symmetry plane. How much is this potential, and why? What happens if we put an infinite conductive plate of an infinitesimal thickness in this plane? This slightly upgraded model can be used to estimate E-fields and potential underneath an overhead high voltage power line. The excessive level of them is significant public concern about adverse health effects.
9. Ask the instructor for additional questions.

More details can be found in [http://bourabai.kz/assis/Revista-Bras-Ens-Fis-V21-p469-475\(1999\).pdf](http://bourabai.kz/assis/Revista-Bras-Ens-Fis-V21-p469-475(1999).pdf)

Problem 11. Uniformly Charged Sheet. Consider a uniformly charged a finite rectangular ($a \times b$) sheet carrying a positive charge Q that spreads uniformly over the area with a surface density $\rho_{eA} = Q/ab$ [C/m²]. Sketch out the equipotential lines of potential and E-force on some plane above the sheet ($Z = \text{const.}$). Then copy, paste, and run the following Matlab script that calculates and plots the lines of equal potential and E-force on the plane $Z \leq 10\text{m}$, i.e., above the sheet, starting, for example, from $Z = 0.1\text{m}$. Explain differences if you found them.



```

clc; clear;
a=input('Enter the length of square side a<=1m: '); b=input('Enter the length of square side b<=1m: ');
Z=input('Field plane location Z<=10m: '); sum=0;N=121; NN=250;
x=linspace(-5*a,5*a,NN); y=linspace(-5*b,5*b,NN); [X,Y]=meshgrid(x,y);
for jx=1:N; x0=-a/2+(jx-1)*a/(N-1); for jy=1:N; y0=-b/2+(jy-1)*b/(N-1);
R=sqrt((X-x0).^2+(Y-y0).^2+Z^2); Phi=1./R; sum=sum+Phi;
end; end; sum=sum/max(max(sum)); [Ex,Ey]=gradient(sum,a/NN,b/NN);
F1=figure; grid minor; hold on; hLines = streamslice(X,Y,-Ex,-Ey,2);set(hLines,'Color','r','LineWidth',2);
contour(X,Y,sum,20,'LineWidth',2); E=sqrt(Ex.^2+Ey.^2); C=del2(E);
s=meshc(X,Y,-E-Z); zticks([0 Z]); zticklabels({'z = ',num2str(Z)}); view([53,24]); alpha(s,0.1);
xlabel('X-axis [m]'); ylabel('Y-axis [m]'); zlabel('Z-axis [m]');
annotation(F1,'textarrow',[0.52 0.48],[0.45 0.34],'TextLineWidth',2,'FontSize',18,...
'String', {'|E(x,y)| at Z = ',num2str(Z),'m'},'HeadStyle','deltoid','LineWidth',2,'Color','r');

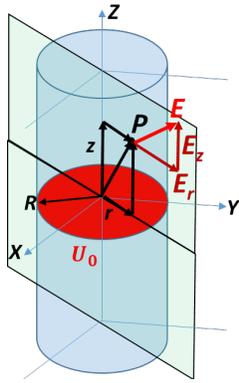
```

1. After the plot appears, retain it on the computer screen by moving it slightly from the center. Then paste the script into Matlab Command Window again and change the plane height to $Z = 1\text{m}$, for example, keeping a and b unaffected. Repeat this procedure several times up to $Z = 10\text{m}$. Compare all plots and clarify the potential and E-field variations in the set.
2. How to explain the persistent peak of potential and E-field intensity at the coordinate origin?
3. What is the orientation of E-fields along the z-axis?
4. Assume that $Z \rightarrow \infty$. Explain why the E-field looks like the E-field of a point charge and $|E| \sim 1/r^2$ where r is the far-away distance from the sheet.
5. Using Gauss's Law (expression (1.23) in Chapter 1), determine and plot the E-field intensity at some distance r in front of an infinite sheet ($a \rightarrow \infty, b \rightarrow \infty$) carrying a uniform surface charge.
6. Ask the instructor for additional questions.

Problem 12. Uniformly Charged Disk. Find the E-field and potential at point P located in the rZ plane due to a **nonconductive** disk carrying the uniformly distributed charge Q^3 . The disk of zero thickness has the radius R . The rotation symmetry means that the E-field image is reproduced identically in any containing z -axis vertical plane (illustrated in light green). Sketch out the 2D equipotential lines of potential and E-force

³ The expressions behind this script can be found on

<https://www.sciencedirect.com/science/article/pii/S0893965911002564#br000040>



produced by the disk in-plane rZ . Then copy, paste, and run the following Matlab script that calculates and plots equal potential and E-force lines. Explain differences if you found them.

1. Describe how to estimate E-field's relative strength and direction (see Section 1.6.3 in Chapter 1) using this plot. Mark area with the highest intensity of E-field.
2. Find the total flux of E- and D-fields flowing through the surface of the sphere surrounding the charged disk.
3. Ask the instructor for additional questions.

```
clc; close all; clear; % Analytical solution
```

```
R=input('Enter the Disk Radius 0 < R [m] < 2: ');
```

```
Q=input('Enter the disk Charge Q [C] = ');
```

```
k=8.9876e+09; Coeff=2*k*Q/R; N=100;
```

```
rho=linspace(-2*R,2*R,N); z=linspace(-2*R,2*R,N); [Rho,Z]=meshgrid(rho,z);
```

```
for i=1:N; for j=1:N;
```

```
fun = @(k0) besseli(0,k0.*Rho(i,j)).*(besseli(1,k0.*R)/k0).*exp(-k0.*abs(Z(i,j)));
```

```
U(i,j)=integral(fun,0,inf); end; end; Ue=Coeff*U; [Er,Ez]=gradient(Ue,4*R/N,4*R/N);
```

```
f3=figure('units','normalized','outerposition',[0 0 1 1]); grid minor; hold on
```

```
hLines = streamslice(Rho,Z,Er,Ez,2);set(hLines,'Color','r','LineWidth',1);
```

```
hContour = contour(Rho,Z,Ue,40); hColorbar = colorbar;
```

```
ylabel(hColorbar,'Electric potential Ue [V]','FontWeight','Bold','FontSize',12);
```

```
xlabel('r-axis [m]'); ylabel('z-axis [m]'); r0=linspace(-R,R,200); plot(r0,0*r0,'k'); axis tight; axis square
```

```
annotation(f3,'textarrow',[0.42 0.48],[0.42 0.51],'TextLineWidth',2,'FontSize',20, ...
```

```
'String', {'Uniformly Charged Disk'}, 'HeadStyle','deltoid','LineWidth',2,'Color','k'); hold off;
```

```
figure(2);s=mesh(Rho,Z,Ue); xlabel('r-axis','FontSize',14,'FontWeight','bold');
```

```
ylabel('z-axis','FontSize',14,'FontWeight','bold'); zlabel('Potential Ue(r,z)','FontSize',14,'FontWeight','bold')
```

```
title('Electric Potential Distribution in rz-plane','FontSize',12,'FontWeight','bold')
```

Problem 13. The Static Model of a Capacitive Top-hat Element of the Current Radiator (see Figure 4.2.1 in Chapter 4). Find the E-field and potential at point P located in the rZ plane due to a perfectly **conducting** disk (grasp the image above) at *absolute* potential $U_0 = 1$ (see Section 1.6.2 in Chapter 1 and Section 2.2.7 in Chapter 3). The disk of zero thickness has the radius R . The rotation symmetry means that the E-field image is reproduced identically in any containing z -axis vertical plane (illustrated in light green). Sketch out the 2D equipotential lines of potential and E-force produced by the disk in-plane rZ . Then copy, paste, and run the following Matlab script that calculates and plots the lines of equal potential and E-force (shown in red)⁴.

```
clc; close all; clear; %Analytical Solution of Laplace Equation in the oblate spheroidal coordinate
```

```
R=input('Enter the Disk Radius 0 < R [m] < 2: '); N=120;
```

```
theta=linspace(0,pi/2,N); eta=linspace(1e-2,log(acosh(4)),N);
```

```
[Eta,Theta]=meshgrid(eta,theta); r=R*cosh(Eta).*sin(Theta); z=R*sinh(Eta).*cos(Theta);
```

```
Ue=-i*imag(acsc(sinh(Eta))); U0=max(max(abs(Ue))); Ue=Ue/U0;
```

```
E1=gradient(Ue); Ez=E1.*cosh(Eta).*cos(Theta); Er=E1.*sinh(Eta).*sin(Theta); C=8*8.8542e-12*R;
```

```
f1=figure('units','normalized','outerposition',[0 0 1 1]); grid minor; hold on
```

```
contour(r,z,Ue,40); contour(-r,z,Ue,40); contour(r,-z,Ue,40); contour(-r,-z,Ue,40);
```

```
hColorbar = colorbar; ylabel(hColorbar,'Electric potential Ue','FontWeight','Bold','FontSize',12);
```

```
xlabel('r-axis [m]'); ylabel('z-axis [m]'); r0=linspace(-R,R,200); plot(r0,0*r0,'k'); axis tight;
```

```
annotation(f1,'textarrow',[0.42 0.48],[0.42 0.51],'TextLineWidth',2,'FontSize',20, ...
```

```
'String', {'Conducting Disk'}, 'HeadStyle','deltoid','LineWidth',2,'Color','k'); axis square
```

```
text(-R,max(max(z))/2,['Disk Capacitance = ',num2str(C),'[F]'],'Color','k','FontSize',24)
```

```
Lines = quiver(r,z,-Er,-Ez,2);set(Lines,'Color','r','LineWidth',1);
```

```
Lines = quiver(-r,z,Er,-Ez,2);set(Lines,'Color','r','LineWidth',1);
```

```
Lines = quiver(r,-z,-Er,Ez,2);set(Lines,'Color','r','LineWidth',1);
```

```
Lines = quiver(-r,-z,Er,Ez,2);set(Lines,'Color','r','LineWidth',1); hold off;
```

```
figure(2);hold on; s=meshc(r,z,Ue); s1=meshc(-r,z,Ue); xlabel('r-axis','FontSize',14,'FontWeight','bold');
```

```
hColorbar = colorbar; view(-23,10); ylabel('z-axis','FontSize',14,'FontWeight','bold'); grid minor
```

```
zlabel('Potential Ue(r,z)','FontSize',14,'FontWeight','bold')
```

```
title('Electric Potential Distribution in rz-plane','FontSize',12,'FontWeight','bold');
```

⁴ The expressions behind this script can be found on <http://physics.usask.ca/~hirose/p812/notes/Ch2.pdf>

1. Explain differences if you found them. Describe the impact of the conductive disk on the potential and E-field patterns, making them different from the plotted ones in problem 1.6.11.
2. Why is a conductive surface always equipotential in static fields?
3. Explain the fringing effect, i.e., the bending and high concentration of the E-force lines near the edge of the disk.
4. Check the boundary conditions in Section 2.3.5 of Chapter 2 and sketch the free charge ρ_{eA} distribution on the disk surface and make sure that it is not uniform as it was assumed in problem 1.16E.
5. Could we expect and where Corona or electrical air breakdown as soon as the disk potential, as well as E-fields, become too high?
6. May you offer the disk design with the edge effect reduction after reading the information in Section 3.3.3 of Chapter 3? Look at the practical solution in https://quickfield.com/advanced/rogowski_electrodes.htm
7. On the top of the E-field plot, you can find the disk capacitance calculated using the simulation data and expressions (3.25), (3.26) from Chapter 3. How does this capacitance change if the fringe fields are omitted?
8. Read Sections 3.3.2 and 3.3.3 in Chapter 3 for more details.
9. Ask the instructor for additional questions.

Problem 14. 3D Capacitor with Circular Plates. A capacitor comprises two parallel conductive disks of the same radius R equally and oppositely charged to *absolute* potentials $+U_0$ and $-U_0$. They are of an infinitesimal thickness and placed a distance d apart. The rotation symmetry means that the E-field structure is reproduced identically in any containing z -axis vertical plane. Sketch out the 2D equipotential lines of potential and E-force produced by the disks in one of these planes. Then copy, paste, and run the following Matlab script⁵ (see next page). Figure 1 demonstrates the electric potential U_e as the equipotential contour curves and E-force as the red arrowed lines. Figure 2 is a 3D image of the potential distribution. Explain differences if you found them.

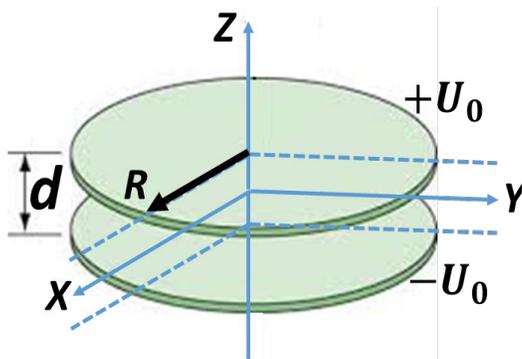
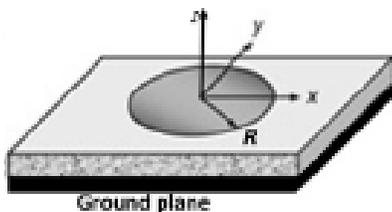


Figure 1 demonstrates the electric potential U_e as the equipotential contour curves and E-force as the red arrowed lines. Figure 2 is a 3D image of the potential distribution. Explain differences if you found them.

1. How to clarify the potential and E-field deviations nearby the plate edges and its almost uniform distribution between the plates?
2. Comment the reason for E-field singularity (enormous increase E-field) around the edges (check Section 3.3.2 in Chapter 3 and Table 3.1 there).
3. Evaluate the potential value along the symmetry line (black dotted) in the first figure and the pink symmetry plane in the second. How much is this potential, and why?



4. What happens if we put an infinite conductive plate of an infinitesimal thickness in this plane?
5. Draw a 3D image of the disk above this ground plate and get the capacitor's static model in the printed circuit board (PCB) or electrically small patch antenna shown on the left.
6. Run the script for $d/R \geq 2$ and comment results. Compare electric potential and E-force line structure with the structure generated by two point charges in problem 1.6E.

7. Explain the difference between the two values of capacitance printed on the top of Figure 1.
8. Ask the instructor for additional questions.

```
clc; close all; clear; fprintf(2, '\nAll coordinates are normalized to disk radius R\n');
fprintf(2, '\nThe normalized to R Gap d/R between disks must be chosen as\n');
fprintf(2, '\nd/R = 0.1 - 1 with step 0.1 or d/R = 2,5,10\n'); d=input('\nEnter the ratio d/R = ');
NN=1.2e2; R=1; r=linspace(0,2*R,NN); z=linspace(-4*d,4*d,NN); [RR,ZZ] = meshgrid(r,z);
```

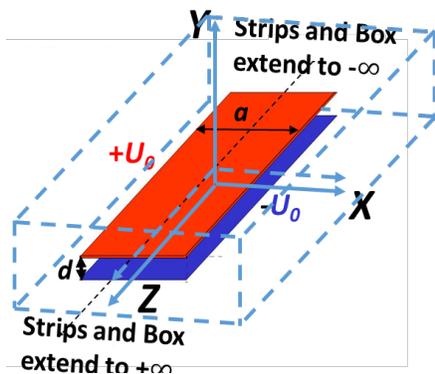
⁵ Great gratitude to Dr. Gradimir V. Milovanovic for giving permission to use the results of the publication <http://www.mi.sanu.ac.rs/~gvm/radovi/gvm-dule1.pdf> and providing additional data included in the script.

```

if d==0.1; P=@(x) (11.170089-4.690523*x.^2-1.474409*x.^4+0.664140*x.^6-1.53822237*x.^8); end
if d==0.2; P=@(x) (6.0773646-2.373046*x.^2+0.164289*x.^4-1.6199941*x.^6+0.7925024*x.^8); end
if d==0.3; P=@(x) (4.3596859-1.507951*x.^2-0.002886*x.^4-0.7929827*x.^6+0.5076101*x.^8); end
if d==0.4; P=@(x) (3.4935288-1.072782*x.^2-0.118818*x.^4-0.2410555*x.^6+0.2211096*x.^8); end
if d==0.5; P=@(x) (2.97067-0.819293*x.^2-0.124922*x.^4-0.0114976*x.^6+0.077654494*x.^8); end
if d==0.6; P=@(x) (2.6206072-0.653164*x.^2-0.088666*x.^4+0.0576722*x.^6+0.0184320*x.^8); end
if d==0.7; P=@(x) (2.3699300-0.533814*x.^2-0.048402*x.^4+0.0639551*x.^6-0.0022945*x.^8); end
if d==0.8; P=@(x) (2.1817877-0.442585*x.^2-0.017287*x.^4+0.0511089*x.^6-0.0075254*x.^8); end
if d==0.9; P=@(x) (2.0356215-0.370213*x.^2+0.002996*x.^4+0.0358529*x.^6-0.0073136*x.^8); end
if d==1.0; P=@(x) (1.91903-0.311595*x.^2+0.014564*x.^4+0.0233527*x.^6-0.005655492*x.^8); end
if d==2.0; P=@(x) (1.41007-0.071116*x.^2+0.008674*x.^4-0.0005066*x.^6-0.0000309361*x.^8); end
if d==5; P=@(x) (1.14348-0.005373*x.^2+0.000182*x.^4); end
if d==10; P=@(x) (1.06773-0.000666*x.^2+0.000000*x.^4); end; for ii=1:length(r); for jj=1:length(z);
VV=@(x) (1./sqrt(RR(ii,jj)^2+(ZZ(ii,jj)-d/2+1i*x).^2)-1./sqrt(RR(ii,jj)^2+(ZZ(ii,jj)+d/2+1i*x).^2)).*P(x);
Int(ii,jj)=integral(VV,-1,1); end; end; Ue=real(Int)/pi; [Er Ez]=gradient(Ue,2*R/NN,8*d/NN);
C=8.854e-12*sum(sum(Er.^2+Ez.^2))*(2*R/NN)*(8*d/NN); C0=8.854e-12*pi*R^2/d;
figure('units','normalized','outerposition',[0 0 1 1]); grid minor; hold on;
hLines = streamslice(RR,ZZ,-Er,-Ez,2);set(hLines,'Color','r','LineWidth',1);
hLines = streamslice(-RR,ZZ,Er,-Ez,2);set(hLines,'Color','r','LineWidth',1);
contour(RR,ZZ,Ue,50); contour(-RR,ZZ,Ue,50); hh=colorbar;
ylabel(hh,'Electric Potential Intensity Ue(r/R,z/R)','FontWeight','Bold','FontSize',15);
xlabel('r/R-axis [m]'); ylabel('z/R-axis [m]'); r0=linspace(-R,R,200);
axis 'tight'; rectangle('Position',[-R,-d/2,2*R,4*d/NN],'FaceColor','k');
rectangle('Position',[-R,d/2,2*R,4*d/NN],'FaceColor','k');
text(0,-d/2,'Uo = -1','Color','b','FontSize',50); text(0,1.1*d/2,'Uo = +1','Color','r','FontSize',50)
text(-2*R,4*d,[ 'Capacitance according to (3.26) = ',num2str(C0),' [F]'],'Color','k','FontSize',40)
text(-2*R,3.4*d,[ 'Capacitance according to (3.27) = ',num2str(C),' [F]'],'Color','k','FontSize',40);
if d<2; plot(-2*R:0.1:2*R,0,'*m','LineWidth',4); end; if d>=2; axis square; xlim([-4*d,4*d]); end;
figure(2);hold on; meshc(RR,ZZ,Ue); meshc(-RR,ZZ,Ue); xlabel('r/R-axis','FontSize',14,'FontWeight','bold');
ylabel('z/R-axis','FontSize',14,'FontWeight','bold'); view(84,21);
hh1=colorbar; ylabel(hh1,'Electric Potential Intensity Ue(r/R,z/R)','FontWeight','Bold','FontSize',15);
zlabel('Electric Potential Ue(r/R,z/R)','FontSize',14,'FontWeight','bold');grid minor;
title('Electric Potential Intensity in rz-plane','FontSize',12,'FontWeight','bold')
vert = [-2*R 0 -1;2*R 0 -1;-2*R 0 -1;-2*R 0 1;2*R 0 1;-2*R 0 1;-2*R 0 1];
fac = [1 2 6 5;1 2 6 5;1 2 6 5;1 2 6 5;1 2 6 5;1 2 6 5;1 2 6 5];
ph=patch('Vertices',vert,'Faces',fac,'FaceVertexCData',hsv(6),'FaceColor','flat');alpha(ph,0.2)

```

Problem 15. 3D Parallel Strip Model of Capacitor.



A capacitor comprises two parallel conductive strips of equal width a and oppositely charged to potentials $+U_0$ and $-U_0$ as $U_0 = 100\text{V}$. They are of an infinitesimal thickness and placed a distance $d = 0.6a$ apart. Generally $d \gg a$, but accepting such practically rear ratio as $d/a = 0.6$ allows to demonstrate noticeably the effect of fringe fields (look at Figure 3.1.10 in Chapter 1). The strips are placed into a relatively large metal box to make the numerical simulation manageable short. This code solves the 2D Poisson's equation numerically for electric potential (for more details, read

<https://www.mathworks.com/matlabcentral/fileexchange/42773-electric-field-of-a-parallel-plate-capacitor-using-2d-poisson-equation>). Sketch out your version of 2D equipotential lines of

potential and E-force produced by the strips at XY-plane. Then copy, paste, and run the following Matlab script. The first appeared figure demonstrates the electric potential U_e as the equipotential contour curves and E-force as the red arrowed lines. The second figure is the contour plot of the E-field magnitude, while the third one is a 3D image of the potential distribution.

```

clc; close all; clear; % Poisson's Equation numerical solution
a=input('Input the strip width a<=5m: '); d=input('Input the strip separation d<a: ');
Nx = 101; Ny = 121; mpx=ceil(Nx/2); mpy=ceil(Ny/2); Ni = 750; h=d/a;
V = zeros(Nx,Ny); T = 0; B = 0; L = 0; R = 0; V(1,:) = L; V(Nx,:) = R; V(:,1) = T;
V(1,1) = 0.5*(V(1,2)+V(2,1)); V(Nx,1) = 0.5*(V(Nx-1,1)+V(Nx,2));
V(1,Ny) = 0.5*(V(1,Ny-1)+V(2,Ny)); V(Nx,Ny) = 0.5*(V(Nx,Ny-1)+V(Nx-1,Ny));
l_p = 51; lp = floor(l_p/2);
p_p = ceil(l_p*h/2); pp1 = mpx+p_p; pp2 = mpx-p_p;

```

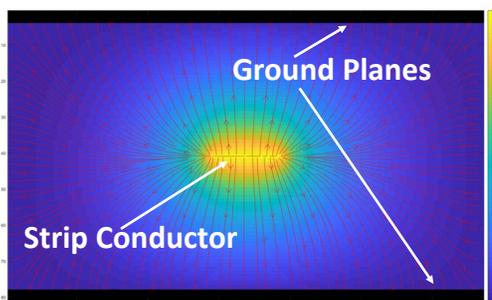
```

for z = 1:Ni; for i=2:Nx-1; for j=2:Ny-1; V(pp1,mpy-lp:mpy+lp) = 100; V(pp2,mpy-lp:mpy+lp) = -100;
V(i,j)=0.25*(V(i+1,j)+V(i-1,j)+V(i,j+1)+V(i,j-1)); end; end; end
V = V'; [Ex,Ey]=gradient(V); Ex=-Ex; Ey=-Ey; E=sqrt(Ex.^2+Ey.^2); x=(1:Nx)-mpx; y=(1:Ny)-mpy;
% Display Potential and E-force Lines
figure('units','normalized','outerposition',[0 0 1 1]);grid minor; hold on;
hLines = streamslice(x,y,Ex,Ey,4);set(hLines,'Color','r','LineWidth',1);
contour(x,y,V,60); h1=colorbar('southoutside','fontsize',12);
xlabel(h1,'Electric potential Ue [V]','FontWeight','Bold','FontSize',20);
title('Electric Potential Ue(x,y)and E-force Lines','fontsize',14); xlabel('x-axis','fontsize',14); ylabel('y-axis','fontsize',14);
rectangle('Position',[-p_p-0.5,-l_p/2,1,l_p],'FaceColor','k')
rectangle('Position',[p_p,-l_p/2,1,l_p],'FaceColor','k'); plot(0,-(Ny-1)/2:1:(Ny-1)/2,'*k');
% Contour Display of E-field
f3=figure(2); contour(x,y,E,120); g2=colorbar('fontsize',12);
ylabel(g2,'E-field Intensity [V/m]','FontWeight','Bold','FontSize',15);
xlabel('x-axis','fontsize',14); ylabel('y-axis','fontsize',14);
h2=title('Electric field distribution, E (x,y) in V/m','fontsize',14);
set(h2,'fontsize',14); movegui(f3,[100,100]); grid minor
% Contour Display of electric potential
figure(3); hh=meshc(x,y,V); axis([-2*lp,2*lp,-2*lp,2*lp]); g3=colorbar('southoutside','fontsize',12);
xlabel(g3,'Electric Potential Ue [V]','FontWeight','Bold','FontSize',15);
ylabel('x-axis','fontsize',14); ylabel('y-axis','fontsize',14); zlabel('Electric Potential [V]','fontsize',14)
h3=title('Electric Potential Distribution, V(x,y) [V]'); set(h3,'fontsize',14); view(-13,32); grid minor
set(hh(2),'ShowText','on','LineWidth',1);
vert = [0 -50 -100;0 50 -100;0 50 -100;0 -50 -100; 0 -50 100;0 50 100;0 50 100;0 -50 100];
fac = [1 2 6 5;1 2 6 5;1 2 6 5;1 2 6 5;1 2 6 5;1 2 6 5];
ph=patch('Vertices',vert,'Faces',fac,'FaceVertexCData',hsv(6),'FaceColor','flat');alpha(ph,0.2)

```

1. Explain differences if you found them.
2. How to describe the potential and E-field deviations nearby the plate edges and their almost uniform distribution between the plates?
3. Comment the reason for E-field singularity (colossal increase E-field) around the edges (check Section 3.3.2 in Chapter 3 and Table 3.1).
4. Evaluate the potential value along the symmetry line (black dotted) in the first figure and the pink symmetry plane in the second. How much is this potential, and why?
5. What happens if we put an infinite conductive plate of an infinitesimal thickness in this plane?
6. Compare the picture in Figure 6.1.1c of Chapter 6 and explain why the E-field pattern in microstrip line shielded by a metal box looks so similar (read Section 6.1.1 about TEM mode).
7. Ask the instructor for additional questions.

Problem 16. Potential and Field Distribution in Microstrip Line (2D Static Model). Stripline is a strip conductor sandwiched by dielectric between a pair of ground planes.



The drawing on the left displays the 2D electromagnetic model of stripline with perfect electrical conductors. The strip conductor's width is typically much smaller than the ground plane's width. Its thickness is negligible in its turn. The relative dielectric constant of the material is assumed 1 (air). In Section 6.1.1 of Chapter 6, you learn that the shown potential field distribution corresponds to TEM-mode propagating and carrying energy in this trendy line printed on a dielectric substrate. The colorbar reflects the intensity of the potential.

Copy and paste Matlab script below into a newly created file in

Editor Window, then save it as StripLineStatic.m, for example. Push Run button  in Editor Window and proceed with simulation following the instructions on the screen.

```

clear all; clc; close;
po=20; n=40; a=7*(po-1)+1; xcord=1:1:a-1; ycord=1:1:2*n; alpha=1.8;
epsilon0=(1/(36*1e+9*pi)); Ey=zeros(2*n,a-1); Ex=zeros(2*n,a-1); V_now=zeros(2*n+1,a);
V_now(n+1,3*(po-1)+1:4*(po-1)+1)=1; V_prev=V_now; V_prev(2,1)=1;
iter=0; while(max(max(abs(V_now-V_prev)))>0.001); iter=iter+1; V_prev=V_now;

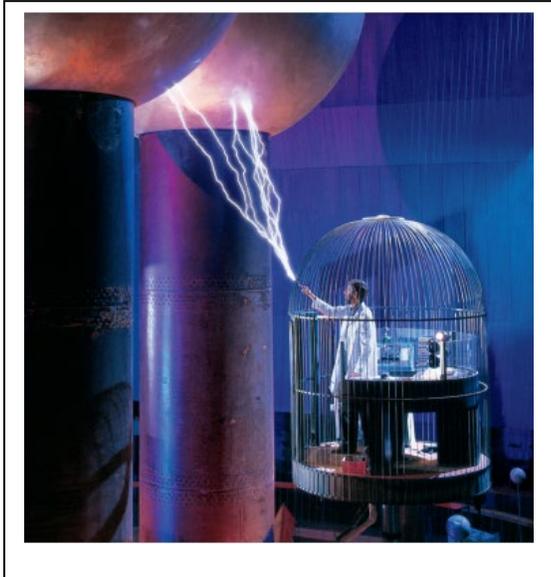
```

```

for i=2:1:2*n; for j=2:1:a-1; if(i<n+1 || i>n+1)
V_now(i,j)=V_now(i,j)+alpha*(0.25*(V_now(i-1,j)+V_now(i+1,j)+V_now(i,j+1)+V_now(i,j-1))-V_now(i,j));
end; if(i==n+1 && (j<3*(po-1)+1 || j>4*(po-1)+1));
V_now(i,j)=V_now(i,j)+alpha*(0.25*(V_now(i-1,j)+V_now(i+1,j)+V_now(i,j+1)+V_now(i,j-1))-V_now(i,j));
end; end; end; end;
V_fin=V_now; imagesc(V_fin); colorbar; hold all
Ey=-0.5*(V_fin(2:2*n+1,1:a-1)+V_fin(2:2*n+1,2:a))-0.5*(V_fin(1:2*n,1:a-1)+V_fin(1:2*n,2:a))*n;
Ex=-0.5*(V_fin(2:2*n+1,2:a)+V_fin(1:2*n,2:a))-0.5*(V_fin(1:2*n,1:a-1)+V_fin(2:2*n+1,1:a-1))*(po-1);
hLines = streamslice(xcord,ycord+0.5,Ex,Ey,3); set(hLines,'Color','r'); grid minor
rectangle('Position',[0,0,140,4],'FaceColor','k'); rectangle('Position',[0,78,140,4],'FaceColor','k')

```

Problem 17. Faraday Cage. "Everybody has heard of the Faraday cage effect, whereby a wire mesh or metal



screen serves to block electric fields and electromagnetic waves. Faraday reported his experiments with a twelve-foot mesh cube in 1836, and engineers and physicists have used metal shielding to isolate circuits and systems ever since. A familiar example is the door of a microwave oven with its metal screen with holes. The screen keeps the microwaves from getting out while allowing light, with its much shorter wavelength, to pass through. Science museums sometimes dramatize the effect with electric sparks, as illustrated in the picture on the left.⁶

Copy and paste Matlab script below into a newly created file in **Editor Window**, then save it as StripLineStatic.m, for example.

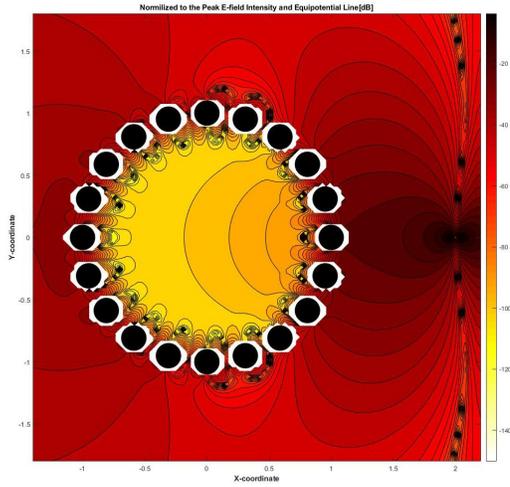
Push Run button  in Editor Window and proceed with simulation following the instructions on the screen.

```

clear; clc; close all
n=input('Enter the Number of wires forming cylindrical cage n = ');
r=input('Enter the wire 0.01 <= Radius <=0.1 [m], r = ');
c = exp(2i*pi*(1:n)/n); rr = r*ones(size(c));
N = max(0,round(4+.5*log10(r))); npts = 3*N+2; circ = exp((1:npts)*2i*pi/npts);
z = []; for j = 1:n; z = [z; c(j)+rr(j)*circ]; end
A = [0; -ones(size(z))]; zs = 2; rhs = [0; -log(abs(z-zs))];
for j = 1:n; A = [A [1; log(abs(z-c(j)))]];
for k = 1:N; zck = (z-c(j)).^(-k);
A = [A [0;real(zck)] [0;imag(zck)]]; end; end
X = A\rhs; e = X(1); X(1) = []; d = X(1:2*N+1:end);
X(1:2*N+1:end) = []; a = X(1:2:end); b = X(2:2:end);
x = linspace(-1.4,2.2,120); y = linspace(-1.8,1.8,120);
[xx,yy] = meshgrid(x,y); zz = xx+1i*yy; uu = log(abs(zz-zs));
for j = 1:n; uu = uu+d(j)*log(abs(zz-c(j)));
for k = 1:N; zck = (zz-c(j)).^(-k); kk = k+(j-1)*N;
uu = uu+a(kk)*real(zck)+b(kk)*imag(zck); end; end
for j = 1:n; uu(abs(zz-c(j))<rr(j)) = NaN; end
z = exp(pi*1i*(-50:50)/50);
for j = 1:n; disk = c(j)+rr(j)*z; fill(real(disk),imag(disk),[0 0 0]);
hold on, plot(disk,'k'); end; gradDB=20*log10(abs(gradient(uu)));
contourf(xx,yy,gradDB-max(gradDB(:),40), colormap(flipud(hot)),
axis([-1.4 2.2 -1.8 1.8]); axis square, plot(real(zs),imag(zs),'r');
colorbar; xlabel('\bfX-coordinate'); ylabel('\bfY-coordinate');
title('\bfNormalized to the Peak E-field Intensity and Equipotential Line[dB]')

```

⁶ S. Jonathan Chapman, David P. Hewett, Lloyd N. Trefethen, Mathematics of the Faraday Cage, SIAM REVIEW, 2015 Society for Industrial and Applied Mathematics, Vol. 57, No. 3, pp. 398–417, https://people.maths.ox.ac.uk/trefethen/chapman_hewett_trefethen.pdf



The image on the left displays the normalized to the peak (located outside the cage) intensity of E-field ($n = 10$, $r = 0.1$) in dB, i.e., $20\log_{10}(\text{abs}(E))$. The white ring around each wire demonstrates that the electric potential on the perfectly conductive wire surface is equal to zero, as required. Depending on the position inside the cage, the cage shielding reaches 110 – 120 dB. Run the Matlab script for different combinations of n and r to investigate the cage shielding efficacy.