

To boost electromagnetics' flavor, we accompanied many problems with short Matlab code acting as a *calculator and result illustrator*. We completely ignored the description of algorithms behind the codes because they are often quite complicated and required ample, very sophisticated as well as abstract mathematical specifics poorly helping to understand the physical picture. Such an approach lets us shift the focus from the often emasculated and practically fruitless problems primarily based on math transformations to more complicated but close to practical tasks. We hope to visualize typically invisible EM fields and stimulate in-depth analysis and the following discussion of the fundamental principles. Projecting images onto a big screen, the lectures may organize whole-class conversation making the audience significantly engaged. These scripts are for people who have never programmed in Matlab or CST before.

We cheer our readers to look through Matlab scripts and CST models to discover and sophisticate the algorithm embedded in them. We encourage you to use the student edition of MATLAB and CST STUDIO SUITE® to get enhanced problem understanding.

Attention. Regrettably, copy and paste into Matlab Command Window saves appropriate Matlab format just in Chrome Web Browser. You have to restart <https://emfieldbook.com/> in <https://www.google.com/> if your browser is different.

Problem 1. Top-hat Dipole. Refresh the material of Sections 4.2.2 and 4.3.1 in Chapter 4. The top-hat radiator location is shown schematically on the left by the black lines. Go to <https://1drv.ms/f/s!AjtsKS-uvNP1avsbUFY9UxsLVVE>, click subsequently on the icon Document, and TopHatDipole, and Download the files top_hat_Frequency.cst and top_hat_Time.cst in a newly created computer file directory named, for example, Top_Hat. Then run top_hat_Frequency.cst project first to get the figures depicted below. Figure 1 illustrates E-force lines nearby the dipole visible inside while the normalized to the peak surface current distribution along the dipole is demonstrated in Figure 2 (blue line) and was generated by DipoleCurrent.m Matlab script using Surface CurrentEdit.txt file (after CST run is finished,

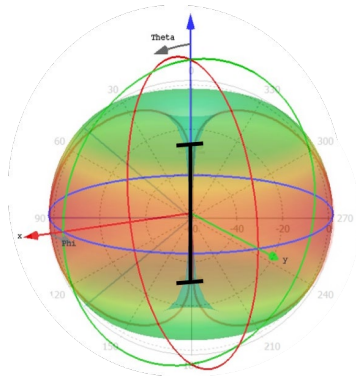


Figure 1

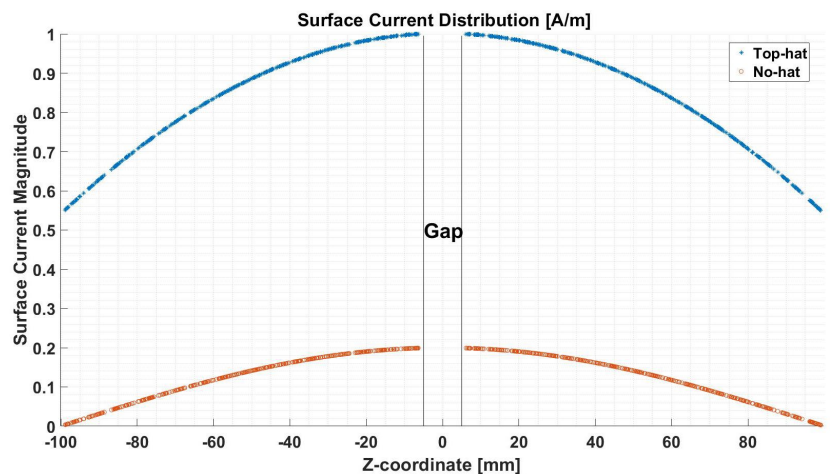
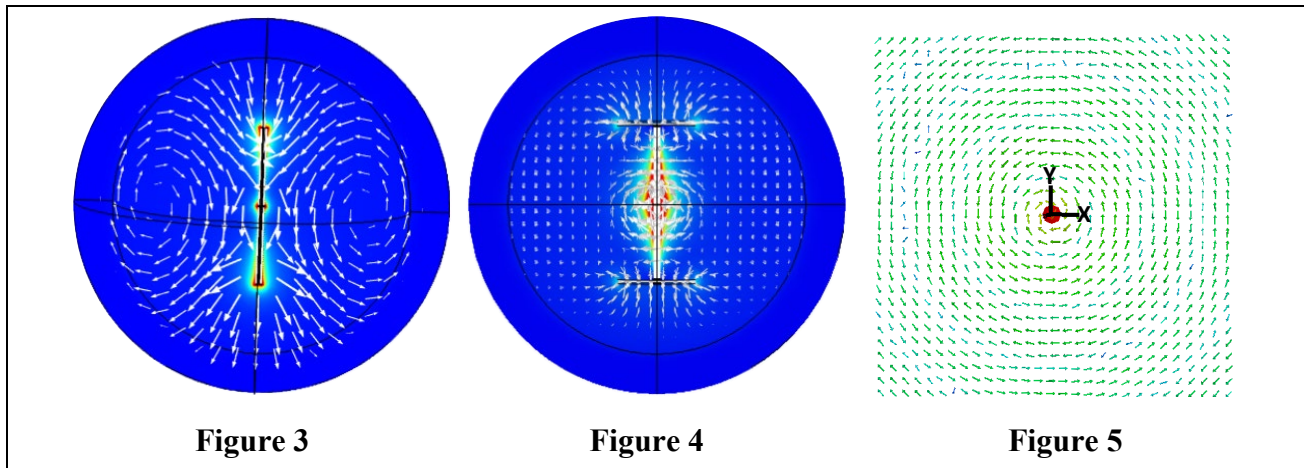


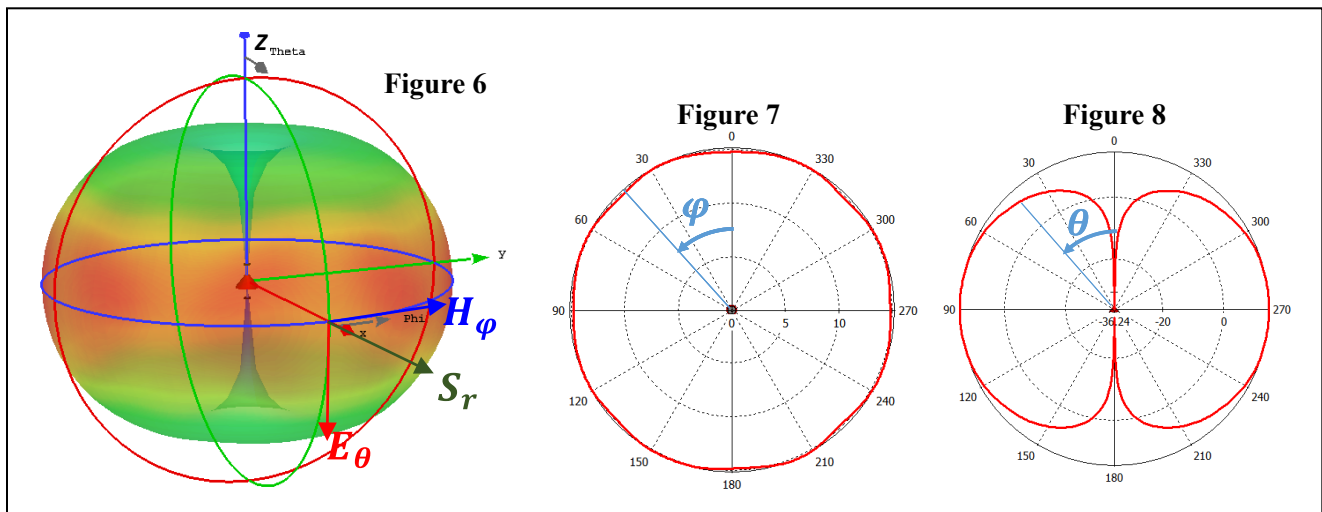
Figure 2

you may get your text file marking the surface current in Navigation Tree, then going to Post Processing Import/Export and choosing Plot Data (ASCII)). The red line of this plot shows the current distribution to compare with no capacitive hats. Pay attention that the simulation frequency 0.5GHz corresponds to wavelength $\lambda = 0.6\text{m}$, while the dipole length $L = 0.1\text{m} = 0.17\lambda$ and is short enough to be a relatively good model of the infinitesimal current element. If you have some concerns, reduce the frequency and rerun the

CST program. Figures 3 and 4 display E-field intensity and E-vectors around the dipole without and with hats, consequentially. **Do the differences match your expectations?** Figure 5 shows H-vectors for both dipoles.



Figures 6 - 8 are the far-field 3D, H-plane, and E-plane radiation patterns (compare with Figure 4.3.2 in Chapter 3). Several animation files are stored in <https://1drv.ms/f/s!AjtsKS-uvNP1avsbUFY9UsxLVVE>



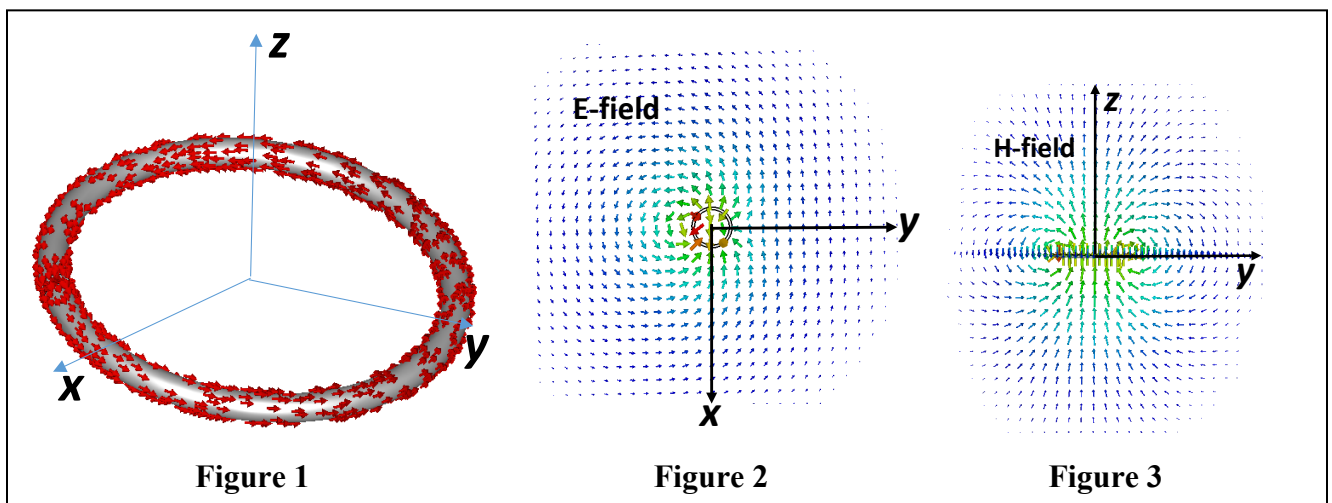
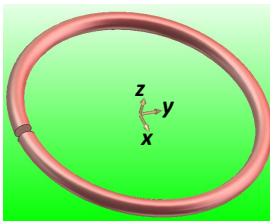
Part of them was produced by the much longer running top_hat_dipole_Time.cst model.

It's time to check your understanding of CST simulation data.

1. What is the surface current distribution along with an ideal current source radiator and such current unit dimension? What is the meaning of the parameter k_1 and k_2 in expression (4.60) of Chapter 4?
2. Running E_vector_Anim2.gif, E_field_Anim2.gif, H_vector_Anim.gif, and H-vector_Anim_2.gif verify the image in Figure 4.2.3 on page 180 (please correct the wrongly specified distance λ to $\lambda/2$). What is the separation between nearby green circles (H-vector) and blue circles (E-vector) in these pictures (*Hint*. Use the expressions (4.61) - (4.64) in Chapter 4)?
3. Looking at Figure 2, explain why a top-hat dipole is a better model of an elementary electric radiator.
4. Is it possible to make some suggestions about the far-field wavefront shape from these animations? What are the differences between spherical and plane wavefront? Could you propose some ideas about how to develop a perfect far-field plane, cylindrical or spherical wavefront?
5. Run top_hat_dipole_EfieldVector.gif and top_hat_dipole_HfieldVector.gif and explain the mutual orientation of near-field E-vector and H-vector and relative to the surface current direction along the dipole.

6. What components of E- and H-fields are visible in near-fields structures? Could we expect the same field structure in the far-field area? If no, explain the differences.
 7. Explain the mutual orientation of E-, H- and S-vectors in Figure 4. How to explain the existence of radiation condition (3.72) in Chapter 3 and the fact that Poynting's vector $\mathbf{S} \sim \mathbf{r}_0/r^2$? What is the unit dimension of Poynting's vector and its physical essence? What is its orientation to the wavefront? Let us assume that the E- and H-vector has shown in Fig. 4 correspond to the time moment $t = 0$. Draw these vectors for $t = T/2$, where T is the wave period.
 8. Why does expression (4.70) symbolize the uniqueness of Maxwell's equations solution?
 9. How to estimate the net power emitted by dipole? Checking expression (4.72) specify parameters affecting the level of this power. What is the preferable way to increase this power?
 10. Explain the 2D patterns displayed in Figures 5 and 6 above and their relationship with the 3D image in Figure 4. Which one corresponds to H-plane or E-plane, vertical/elevation, or horizontal/azimuthal? May we always interpret the H-plane as a horizontal one? (*Hint*. Turn the dipole 90° around y- or x-axis)
 11. Can we include an elementary electric dipole in the class of directional antennas? What does it mean for the antenna to be directional?
 12. Looking at Figures 4 - 6 predict the relative signal magnitude transmitted or received at $\theta = 0^\circ$ and $\theta = 90^\circ$. How is Poynting's vector oriented relative to the far-field waveform?
 13. Could you estimate based on E- and H-fields configuration (see Figure 1 and 3) and simulation data what an inequity is correct for the nearby field, $W_e \gg W_m$ or $W_m \gg W_e$? Go back to Sections 3.1.10 – 3.1.14 in Chapter 3 and explain what kind of reactance (inductive or capacitive) is expected for this dipole. Does your answer match CST data (Check the real and imaginary part of Z11 in 1D Results/Z Matrix of Navigation Tree)?
- Explore more plots from the CST model run, formulate, and answer additional questions. Discuss the CST results with the instructor and team members.

Problem 2. Electrically Small Current Loop. Refresh the material of Sections 4.3.2 and 4.3.3 in Chapter 4. The PEC circular loop of 17mm outer diameter is shown on the left. A discrete port delivering the power of 0.5W and supporting RF electrical current in the loop is located inside the narrow gap. Go to <https://1drv.ms/f/s!AjtsKS-uvNP1avsbUFY9UsxLVVE>, click subsequently on the icon Document, and Loop to download in your computer two files LoopFrequency.cst (Frequency-domain simulation) and LoopTime.cst (Time-domain simulation). Expect a longer running time and higher memory requests for time-domain simulation. Some plots you need to generate are depicted below.



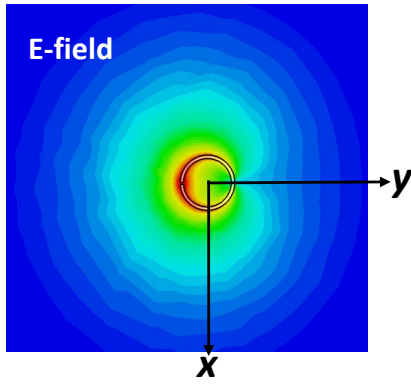


Figure 4

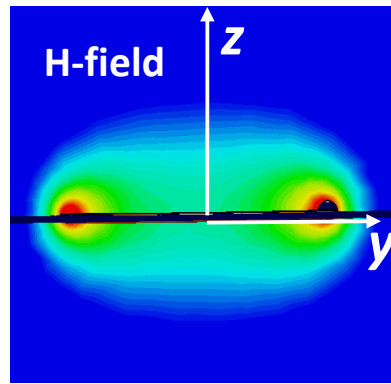


Figure 5

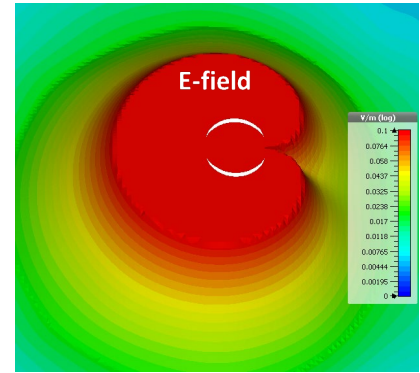


Figure 6

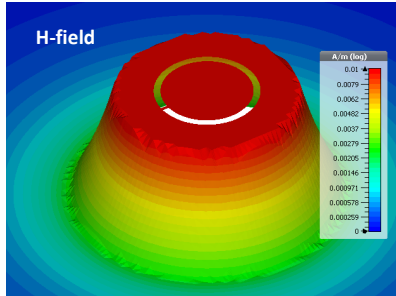


Figure 7

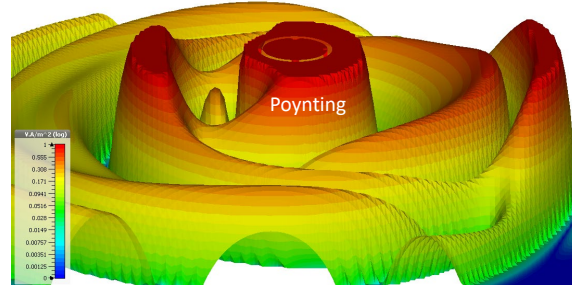


Figure 8

First of all, make sure the current flows along the loop, as Figure 1 illustrates. Then generate the E- and H-vector structures, as shown in Figures 2 and 3, respectively. The next step is to produce E- and H-field strength in cross-sections around the loop as Figures 4 and 5 demonstrate, while Figures 6 and 7 recreate the same through the 3D surface (named carper in CST) plots where the color gradient is proportional to field magnitude. Figure 8 is the power flow (Pointing) intensity in $[W/m^2]$ at some moment in time.

The last drawings in Figures 9 – 11 characterize the loop antenna performance in the far-field area.

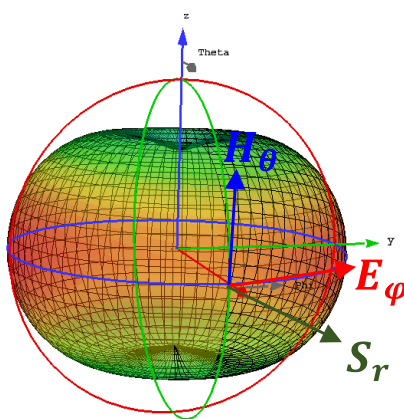


Figure 9

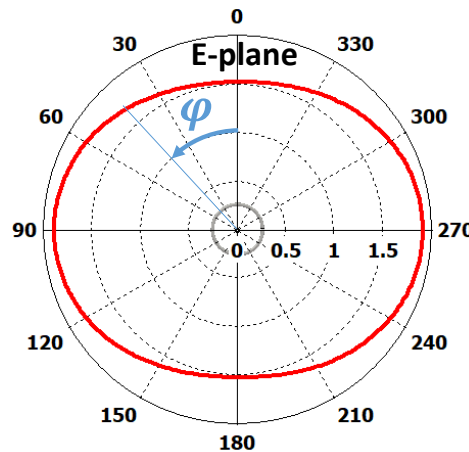


Figure 10

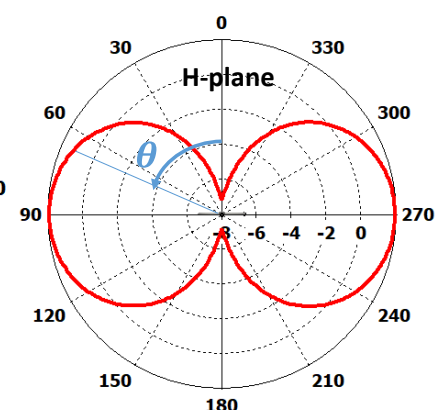


Figure 11

Several useful field animations can be found in <https://1drv.ms/f/s!AjtsKS-uvNP1avsbUFY9UsxLVVE>. We encourage our readers to continue experimenting with CST models to get more results and impressive images.

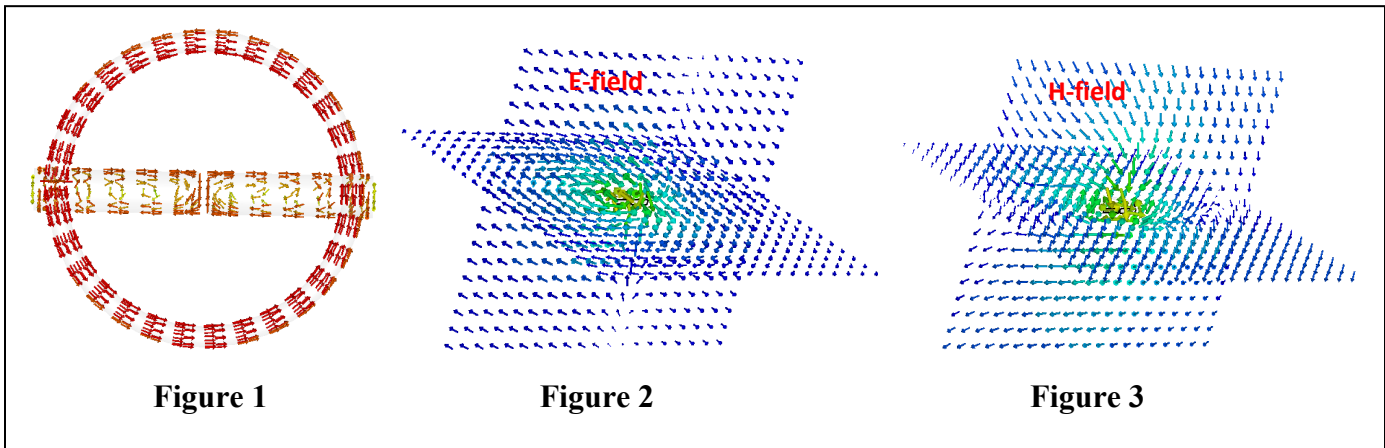
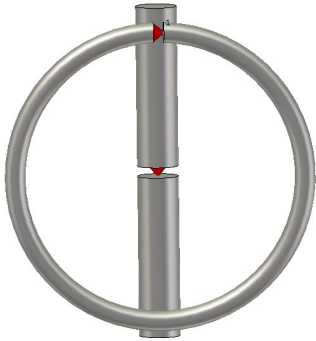
Particularly, investigate the influence of the loop diameter and gap width relative to wavelength on its field structures and radiation patterns. Note that the output time-domain data contain multiple far-field pattern cuts at frequencies between 0.5GHz and 9.5GHz. Just go to Farfield/Excitation in Navigation Tree and choose by double-clicking the desirable frequency cut. It looks surprising how single and so simple an antenna can develop a wide diversity of pattern shapes, although not all are practical. *Could you explain why the patterns vary so strongly as the frequency changes* (difficult question)?

1. Assume there are three loops with a diameter of 0.1m, 1m, and 3m. Which one is electrically small at frequency 10MHz? Clear the definition of an electrically small antenna.
2. Does the direction and magnitude of electric current in Figure 1 correspond to an electrically small loop? Check the animation Loop_Current.gif at <https://1drv.ms/f/s!AjtsKS-uvNP1avsbUFY9UsxLVVE> to be sure.
3. Where the radiation null of the loop pattern is directed, and why? Do the plots in Figures 9 – 11 reflect this correctly? How to use this effect in practice?
4. According to Figure 1, the electric current flows opposite directions through the loop segments touching the same diameter. If so, explain the peak of the radiation pattern at $\theta = 0^\circ$ and 90° instead of null.
5. Does E-vector distribution in Figures 2 and 4 correspond to conservative or non-conservative (curled) field? Give a detailed answer explaining how it happens that E-fields are tangential to the loop surface (*Hint*. Cut the loop into multiple sections with uniform current and consider each part as elementary electric dipole emitting E-field like shown in Figure 1 of the previous problem).
6. Does H-vector distribution in Figures 3 and 5 correspond to conservative or non-conservative (curled) field? Give a detailed answer explaining why H-fields are perpendicular to the loop plane while the part of its force lines is twisted around the loop wire. What is the mutual orientation of E- and H-fields?
7. What components of E- and H-fields are visible in near-fields structures? Is the red field area in Figures 6 and 7 mostly filled by active or reactive energy? Could we expect the same field structure in the far-field area? If no, explain the differences.
8. Could you estimate based on E- and H-fields configuration (see Figure 1 and 3) and simulation data what an inequity is correct for the nearby field, $W_e \gg W_m$ or $W_m \gg W_e$? Go back to Sections 3.1.10 – 3.1.14 in Chapter 3 and explain what kind of reactance (inductive or capacitive) is expected for a small loop. Does the answer match CST data (Check the real and imaginary part of $Z_{1,1}$ in 1D Results/Z Matrix of Navigation Tree)? How to explain the resonance around 1.4GHz and capacitive reactance at higher frequencies (Check Section 3.2.4)?
9. Running Loop_E_Anim.gif, Loop_H_Anim.gif, and Loop_Power_Anim.gif verify the image in Figure 4.2.3 on page 180. What is the separation between nearby green circles (H-vector) and blue circles (E-vector) in these pictures (*Hint*. Use the expressions (4.61) - (4.64) in Chapter 4)? Could you explain why the appearance of E- and H-fields are not synchronized in time, linking your answer with the previous question?
10. Is it possible to make some suggestions about the far-field wavefront shape from these animations? What are the differences between spherical and plane wavefront? Could you propose some ideas about how to develop a perfect far-field plane wavefront?
11. Explain the 2D patterns displayed in Figures 10 and 11 and their relationship with the 3D image in Figure 9. Which one corresponds to H-plane or E-plane, vertical/elevation, or horizontal/azimuthal? May we always interpret the H-plane as a vertical one? (*Hint*. Turn the loop 90° around y- or x-axis)
12. Can we include an elementary loop in the class of directional antennas? What does it mean for the antenna to be directional?
13. Looking at Figures 9 - 11 predict the relative signal magnitude transmitted or received at $\theta = 0^\circ$ and $\theta = 90^\circ$. How is Poynting's vector oriented relative to the far-field waveform?
14. How to explain that the E-plane pattern is an oval and not a circle as expected from expression (4.81) in Chapter 4? *Hint*. Include into your consideration the EM field radiation from the gap and check the clear asymmetrical structures in Figures 4, 6, and 8)?

15. Can a small loop antenna be considered as an effective radiator? How to improve radiation characteristics? Match with a top-hat dipole of comparable geometrical sizes.

Explore more plots from the CST model run, formulate, and answer additional questions. Discuss the CST results with the instructor and team members.

Problem 3. Huygens' Radiator. Refresh the material of Sections 4.3.4 in Chapter 4. CST model of Huygens' radiator was developed as simultaneously radiating orthogonal electric (vertical rod) and magnetic dipole (loop) shown on the left. The radiators were excited according to expression (4.86) on page 190, i.e. $I_e^{exc} = 1 [A]$ and $I_m^{exc} = -Z_0 I_e^{exc} = -120\pi [V]$ to get the cardioid shaped directional pattern. The red dots indicate the location of the current and voltage source. Go to <https://1drv.ms/f/s!AjtsKS-uvNP1avsbUFY9UsxLVVE>, click subsequently on the icon Document, and HuygensRadiator to download in your computer file HuygensTime.cst (Time-domain simulation). Be patient, expecting a relatively long running time (about half an hour on my desktop) and high memory requests. Some plots you need to generate are depicted



below.

First of all, make sure the current flows along the loop, as Figure 1 illustrates. Then generate the E- and H-vector structures, as shown in Figures 2 and 3, respectively. The next step is to produce E- and H-field strength in cross-sections around the loop and dipole as Figures 4 and 5 demonstrate, while Figures 6 and 7 recreate the same through the 3D surface (named carper in CST) plots where the color gradient is proportional to field magnitude. Figure 8 is the power flow (Pointing) intensity in $[W/m^2]$ at some moment in time.

- 1.** Does the direction and magnitude of electric current in Figure 1 correspond to an electrically small antenna? Check the animation Huygens_Current.gif at <https://1drv.ms/f/s!AjtsKS-uvNP1avsbUFY9UsxLVVE> to be sure. Explain how it happens that the wavelet source is the sum of the orthogonal electric and magnetic dipole?
- 2.** Does E-vector distribution in Figures 2 and 4 correspond to conservative or non-conservative (curled) field? Give a detailed answer. Explain the differences between these plots and the similar ones for an electric dipole.
- 3.** Does H-vector distribution in Figures 3 and 5 correspond to conservative or non-conservative (curled) field? Give a detailed answer. Explain the differences between these plots and the similar ones for a small loop. What is the mutual orientation of E- and H-fields?
- 4.** What components of E- and H-fields are visible in near-fields structures? Is the red field area in Figures 6 and 7 mostly filled by active or reactive energy? Could we expect the same field structure in the far-field area? If no, explain the differences. Could you clarify the spike of E-field and Poynting's magnitude appearance in Figures 6 and 8 while the structure of H-field energy stays mostly circular?

5. Explain the 2D patterns displayed in Figures 10 and 11 and their relationship with the 3D image in Figure 9. How is Poynting's vector oriented relative to the far-field waveform? Which one corresponds to H-plane or E-plane, vertical/elevation, or horizontal/azimuthal? May we always interpret the H-plane as a vertical one? (Hint. Turn the loop 90° around y- or x-axis).

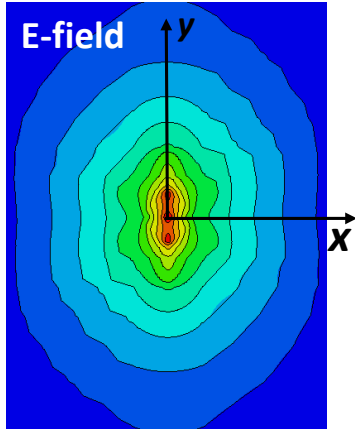


Figure 4

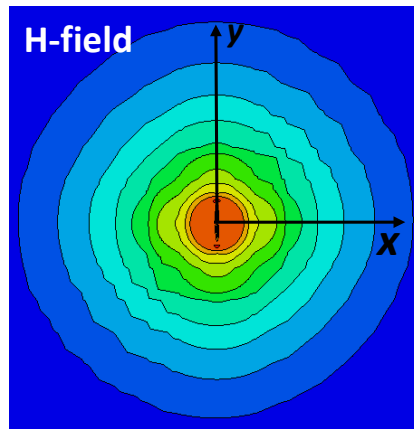


Figure 5

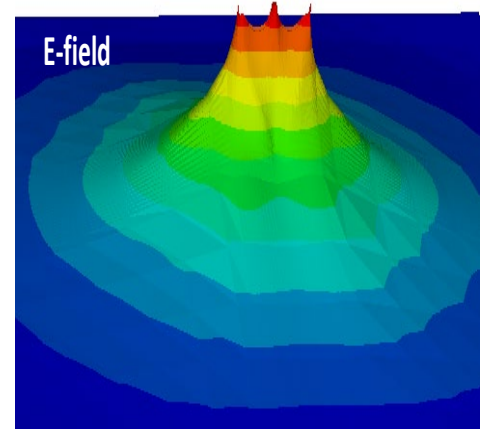


Figure 6

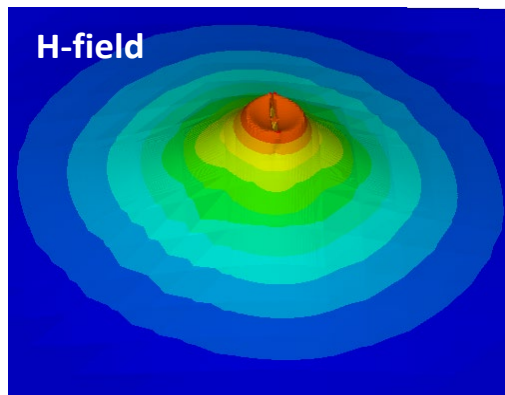


Figure 7

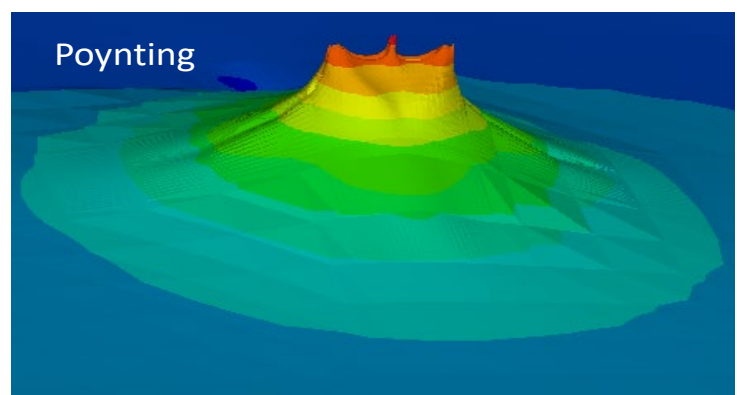
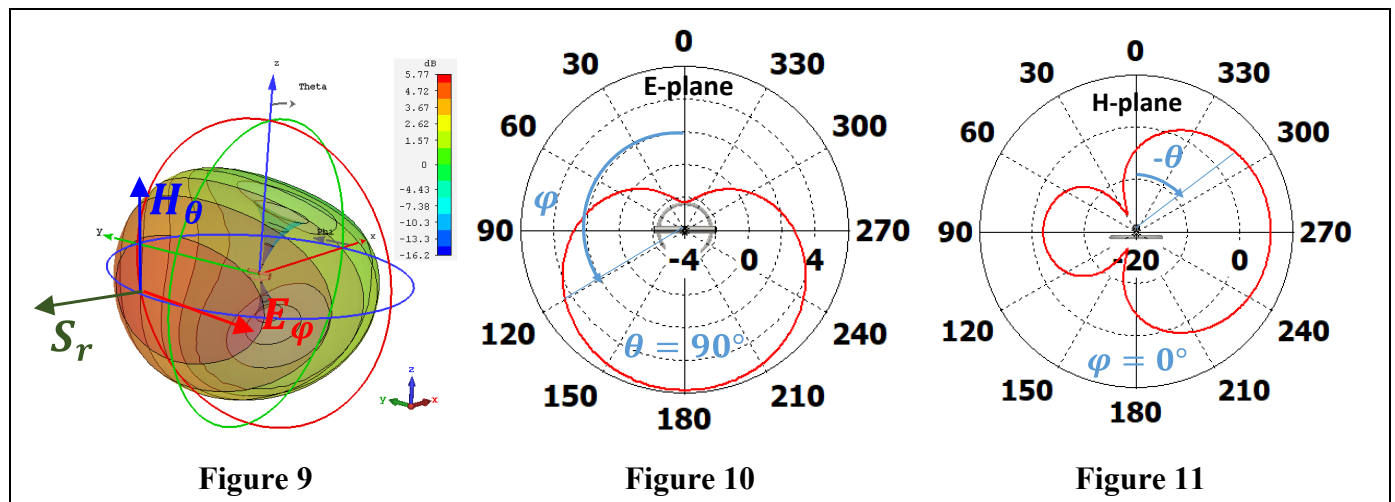
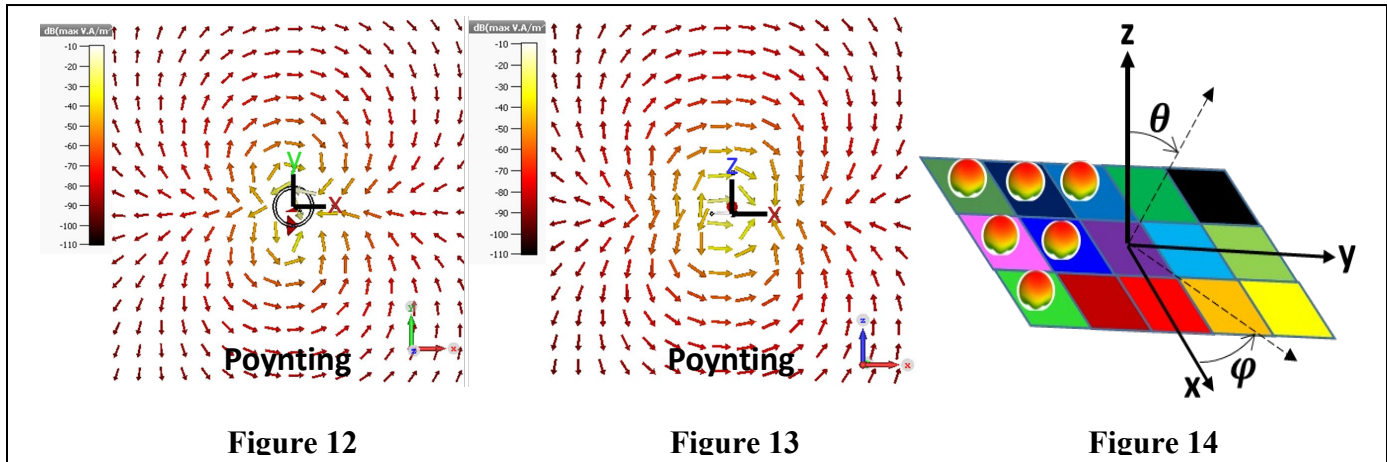


Figure 8

The last drawings in Figures 9 – 11 characterize the loop antenna performance in the far-field area.



6. Compare Figures 9 – 11, as shown in Figure 4.3.9 of Chapter 4, and explain the pattern's cardioid shape formation.
7. Can we include a Huygens' radiator in the class of directional antennas? What does it mean for the antenna to be directional?
8. Describe Huygens' Principle (sometimes called Huygens-Fresnel) and the process of cylindrical/spherical wavefront propagation. What is the shape of the pattern emitted by each wavelet?
9. What is wrong in this Huygens' Principle description: Every point of a wave may be considered the source of secondary wavelets spread out in all directions with speed equal to the wave's speed of propagation.
10. Figures 12 and 13 demonstrate the near-field Poynting's vector orientation in XY- and XZ-plane,

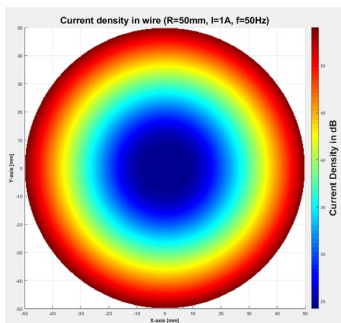



respectively. Do such images reflect the flow of the energy concentrated around (reactive) or leaving (radiated) Huygens' wavelet?

11. Suppose the quilt depicted in Figure 14 is a set of square areas forming a plane far-field wavefront. Treating each such square as Huygens' wavelet with the cardioid pattern (apple shape effigies) explain the wavelet lack of back radiation and propose the algorithm of narrow beam formation (see equations (5.104) – (5.108) in Chapter 5)

Explore more plots from the CST model run, formulate, and answer additional questions. Discuss the CST results with the instructor and team members.

Problem 4. Skin Effect. Refresh the material of Sections 4.4 in Chapter 4. Copy and paste Matlab script



below into Matlab Edit Window and run it pushing  Run-button. Six plots appear on the screen. The full-screen Figure 1 demonstrates the magnitude in dB of volume current density j_{ev} [A/m²] in the copper conductor at 50Hz vs. the wire radius. The actual density is depicted in Figure 2 (bottom center). Figure 3 (top left) illustrates the wire resistance per meter against the frequency. Figure 4 (bottom left) exemplifies the skin depth vs. frequency. Figure 5 (top center) and Figure 6 (top right) are similar to Figure 2 and reflect the current distribution at 1 kHz and 10 kHz, respectively.

1. Might the skin effect be observed in dielectrics with low electrical conductivity?
2. Explain the relationship between EM wave velocity in high conductive materials and the dissipation of energy there.
3. What does it mean that the skin depth is 2mm? How do metal conductivity and frequency impact this parameter? What is the cause that most of EM wave energy and conductivity current concentrate within a thin layer under the conductor's surface?


```

clc; close all; clear; D_wire_m=input('Enter the wire diameter [m] = ');
sigma=input('Enter the conductivity {S/m} =');
u0 = 4*pi*1e-7; ur = 1; rho = 1/sigma; R_m = D_wire_m/2; area = R_m^2*pi; f_circle_Hz = 50; current_A = 1; dr = 200;
[X,Y] = meshgrid(-R_m:R_m/(dr-1):R_m,-R_m:R_m/(dr-1):R_m); r=sqrt(X.^2+Y.^2);
A = sqrt(-1)*2*pi*f_circle_Hz*sigma*u0*ur; [J0] = besseli(0,A.*r);
[J1] = besseli(1,A.*R_m); J_vec = A.*current_A.*(1./(2.*pi.*R_m)).*J0./J1;
J_vec((X.^2+Y.^2)>=R_m^2) = NaN; X(isnan(J_vec)) = NaN; Y(isnan(J_vec)) = NaN;
figure('units','normalized','outerposition',[0 0 1 1]); surf(X*1e3,Y*1e3,20*log10(abs(J_vec)),'LineStyle','none');
hc=colorbar; colormap('jet'); ylabel(hc, '\bfCurrent Density in dB','FontSize',20); axis square; view([0 0 90]);
ht=title(['Current density in wire (R=',num2str(R_m*1e3),'mm, I=',num2str(current_A),'A, f=', num2str(f_circle_Hz), 'Hz)']);
set(ht,'FontSize',20); xlabel('\bfX-axis [mm]'); ylabel('\bfY-axis [mm]')
fff=figure; movegui(fff,'south'); plot(Y(X==0)*1e3, abs(J_vec(X==0))); grid on; xlabel('\bfRadius [mm]'); ylabel('\bfJ [A/m^2]');
title(['\bfCurrent density vs. radius (R=', num2str(R_m*1e3),'mm, I=',num2str(current_A),'A, f=', num2str(f_circle_Hz), 'Hz)']);
Rad = 2 * 1e-3; meu_r = ur; freq = logspace(log10(1),log10(1e7), 100); % [Hz] frequencies
display = 0; [Rdc, Rac] = skin_effect( freq, Rad, sigma, meu_r, display );
f1=figure; movegui(f1,'northwest'); loglog(freq,Rac); hold on; grid minor
loglog(freq,(freq.^0)*Rdc,'k--'); hold off; xlim([freq(1) freq(end)]); legend('AC','DC'); xlabel('\bfFrequency [Hz]');
ylabel('\bfResistance [ohm / m]'); title('\bfAC resistance vs. frequency');
skin_depth = (2./(2*pi*freq*4*pi*1e-7*meu_r*sigma)).^0.5;
f4=figure; movegui(f4,'southwest'); loglog(freq,skin_depth); grid minor;
xlim([freq(1) freq(end)]); xlabel('\bfFrequency [Hz]'); ylabel('\bfSkin Depth [m]'); title('\bfSkin Depth vs. frequency');
freq = 1e3; display = 1; [Rdc,Rac] = skin_effect(freq, Rad,sigma,meu_r,display);
freq = 100e3; display = 2; [Rdc,Rac] = skin_effect(freq,Rad,sigma,meu_r,display);
function [ Rdc, Rac ] = skin_effect(freq,Rad,sigma,meu_r,display)
meu0 = 4*pi*1e-7; meu = meu0 * meu_r; I = 1; Rdc = 1/(sigma*pi*Rad^2);
for ii = 1:length(freq); f = freq(ii); w = 2*pi*f; delta = (2/(w*meu*sigma))^0.5;
k = (1+j)/delta; dr = min(delta,Rad) / 100; minR = max(0, Rad-10*delta);
r = (Rad-dr/2) : -dr : (minR+dr/2); J = (I/(2*pi)) * k^2 / ( exp(-k*Rad) + k*Rad - 1 ) * exp(k*(r-Rad));
absJ = abs(J); Rac_ii = Rdc * (2*pi^2*Rad^2 / I^2) * sum((absJ.^2).*r) * dr;
Rac(ii) = Rac_ii; end; jj = find(freq == 0); Rac(jj) = Rdc;
if (display>=1); Jdc = I/(pi * Rad^2); if (f==0); absJ = Jdc*(r.^0); end
ff=figure; if display==2; movegui(ff,'northeast'); end
plot(r/Rad,absJ,'r-'); grid minor; hold on; plot([0 1],[Jdc Jdc],'k--'); hold off;
xlim([0 1]); ylim([0 max(absJ)*1.2]); xlabel('\bfr / Radius'); ylabel('\bfJ [A / m^2]'); legend('AC','DC');
title_str = strcat('\bfCurrent distribution in conductor at ', '{ ', num2str(f), ' Hz'); title(title_str); end;
end

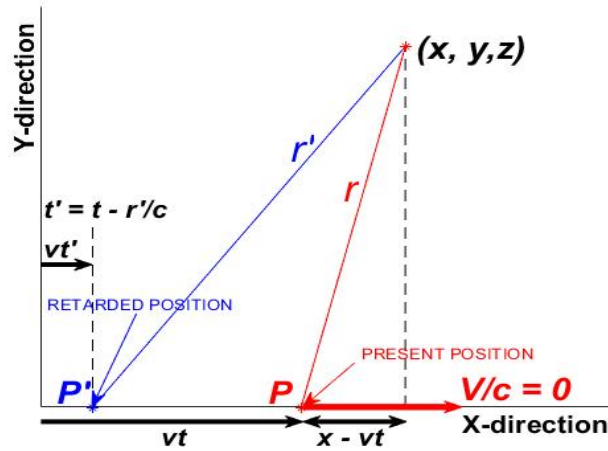
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This Matlab script combines two files published in MathWorks File Exchange, ‘Skineffect Calculation’ by Sven Koerner, and ‘Skin effect solver’ by Yoash Levron. Both files are slightly edited to match the book material.

4. Does an increase in frequency or conductivity diminish or escalate EM energy loss?
5. Using equations (3.11), $\mathbf{j}_{ev} = \sigma \mathbf{E}$, and (4.88) prove that in metals $P_{loss} \sim \sqrt{\sigma}$.
6. Define the meaning of the surface conductivity and explain the differences between surface and volume conductivity. Prove that in metals $P_{loss} \sim \sigma_s$. What is the unit dimension of surface current and conductivity?
7. If the frequency of EM wave propagated in metal is double, does the energy dissipation doubled too?
8. Explain why DC and AC resistance in Figure 3 is not equal.
9. The wire diameter was doubled. Does the energy dissipation per wire unit length increase or drop? Check your answer rerunning the CST model, and pay attention to the current distribution in Figure 2.
10. Explain the current density variations in Figures 5 and 6.
11. How to explain that the graph of skin depth in Figure 4 is a straight line? Can we expect the same results for any frequencies and conductivity (check expression (4.87) in Chapter 4)?
12. How are E- and H-fields oriented about a conductor? Do these fields diffuse into the conductor or stay outside?
13. Do the EM energy carried by the wire propagates outside the wire in the form of EM-wave or concentrates within the wire, moving electrons there?
14. What conducting materials are mostly used for power lines and home wiring?
15. The copper conductors are often silver or gold plated. Why do this if the silver is slightly better while the gold is a worse conductor than copper?
16. Might the wire be hollow?

17. Explain the effect of attenuation increase due to conducting surface roughness.

Problem 5. Charge q Moving with Constant Speed along X-axis.



Please review *The Feynman Lectures on Physics, Volume II, Chapters 20¹, 26²*, or visit MIT OpenCourseWare³, and the material of Section 4.1 in Chapter 4⁴. The following description is limited to the electromagnetic field analysis of a charge q moving along the x -axis with a constant speed v **normalized to the light velocity** c , i.e., $0 \leq v < 1$. Supposedly, the charge is located at the point P at the present moment t shifting from the retarded position P' at the distance $x' = x - vt$. According to equation (26.1)², the scalar φ and only non-zero component A_x of vector potential at the observation point (x, y, z) are

$$\varphi = \frac{q}{4\pi\epsilon_0} \frac{1}{\sqrt{(x-vt)^2 + (1-v^2)(y^2+z^2)}}, \quad A_x = \frac{q}{4\pi\epsilon_0} \frac{v}{\sqrt{(x-vt)^2 + (1-v^2)(y^2+z^2)}} = v\varphi$$

Then the electrical and magnetic fields can be get¹⁻³

$$\mathbf{E} = -\nabla\varphi - \partial\mathbf{A}/\partial t, \quad \mathbf{B} = \nabla \times \mathbf{A}$$

Here r is the distance between the point of observation (point P) and the charge, while $V/c = x_0 v$ is the charge velocity vector. The final expressions are

$$E_x = \frac{q}{4\pi\epsilon_0} \frac{(x-vt)(1-v^2)}{((x-vt)^2 + (1-v^2)(y^2+z^2))^{3/2}}, \quad E_y = \frac{q}{4\pi\epsilon_0} \frac{y(1-v^2)}{((x-vt)^2 + (1-v^2)(y^2+z^2))^{3/2}}$$

$$E_z = \frac{q}{4\pi\epsilon_0} \frac{z(1-v^2)}{((x-vt)^2 + (1-v^2)(y^2+z^2))^{3/2}}$$

$$\mathbf{B} = v\mathbf{x}_0 \times \mathbf{E} = v(y_0 E_z - z_0 E_y) = \frac{q}{4\pi\epsilon_0} \frac{v(1-v^2)(y_0 z - z_0 y)}{((x-vt)^2 + (1-v^2)(y^2+z^2))^{3/2}}$$

¹ https://www.feynmanlectures.caltech.edu/II_20.html

² https://www.feynmanlectures.caltech.edu/II_26.html

³ <https://ocw.mit.edu/courses/nuclear-engineering/22-105-electromagnetic-interactions-fall-2005/readings/chap4.pdf>

⁴ Engineering Electrodynamics. House of Maxwell's Electrodynamics

From an *Energy Conservation Equation* for the relativistic EM fields follows⁵ that the vector product

$$\mathbf{u} = (\mathbf{E} \times \mathbf{B})/\mu_0 \quad [\text{W/m}^2]$$

can be identified as *The Energy Flux* due to the EM fields, i.e., *The Poynting Flux*.

The Matlab code appraises several subprograms and generates multiple plots. Since the EM field images are complicated, be patient, expecting a longer running time and higher memory requests for animated plots. Some plots you need to generate are depicted below.

Figure 1 shows a charge q moving along the x -axis and the same as this problem's title. The screenshot of Figure 2 illustrates the ellipsoidal ($v > 0$) shape of an animated isosurface of the scalar ϕ and vector A_x components of the four-potential². The black arrows on the top and bottom depict the principal ellipsoid axes a , b , and their ratio (on the bottom) $a/b \rightarrow 1$ as $v \rightarrow 0$ while $a/b \rightarrow 0$ as $v \rightarrow 1$. Both potentials are squeezed in the direction of the charge motion. The green arrows demonstrate the vector potential orientation and its space distribution (not in scale).

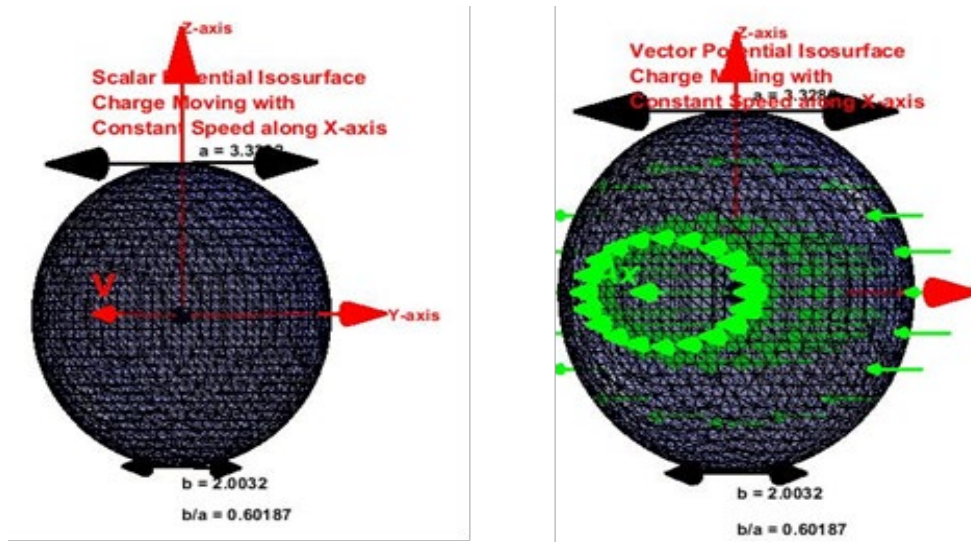
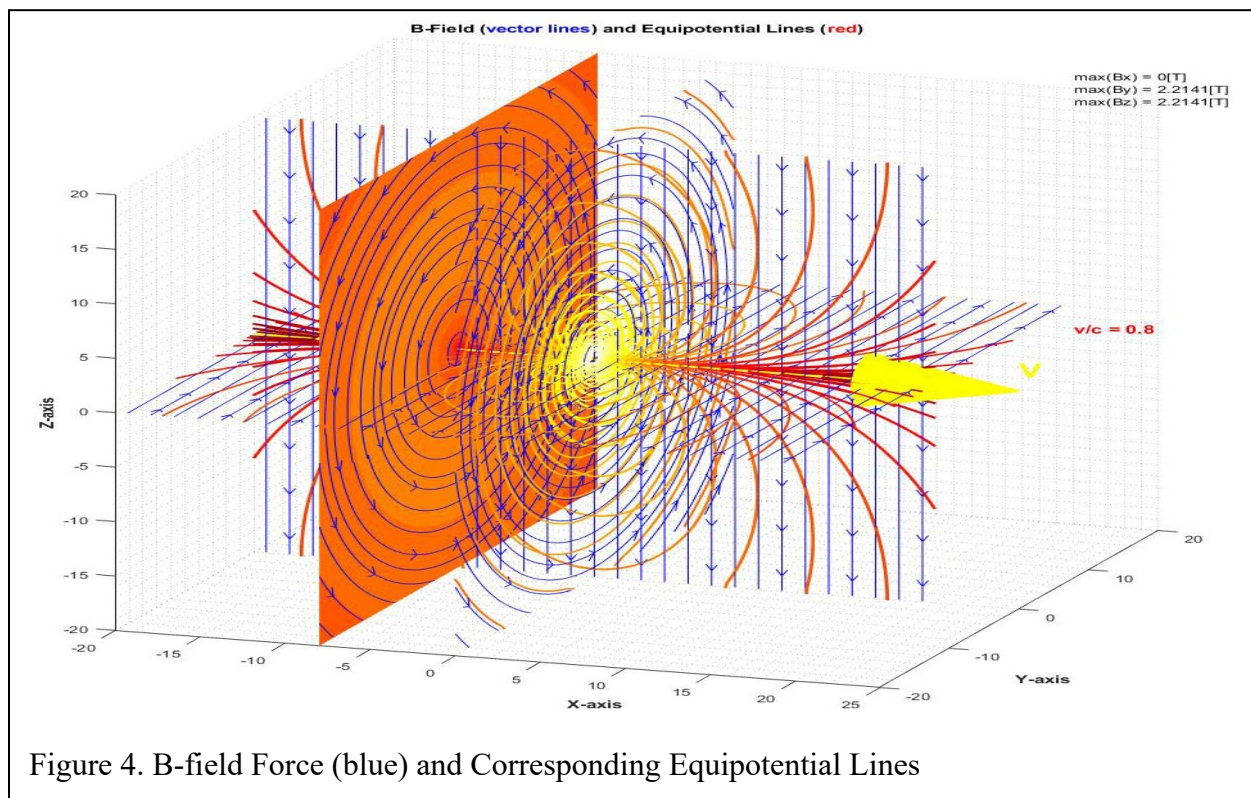
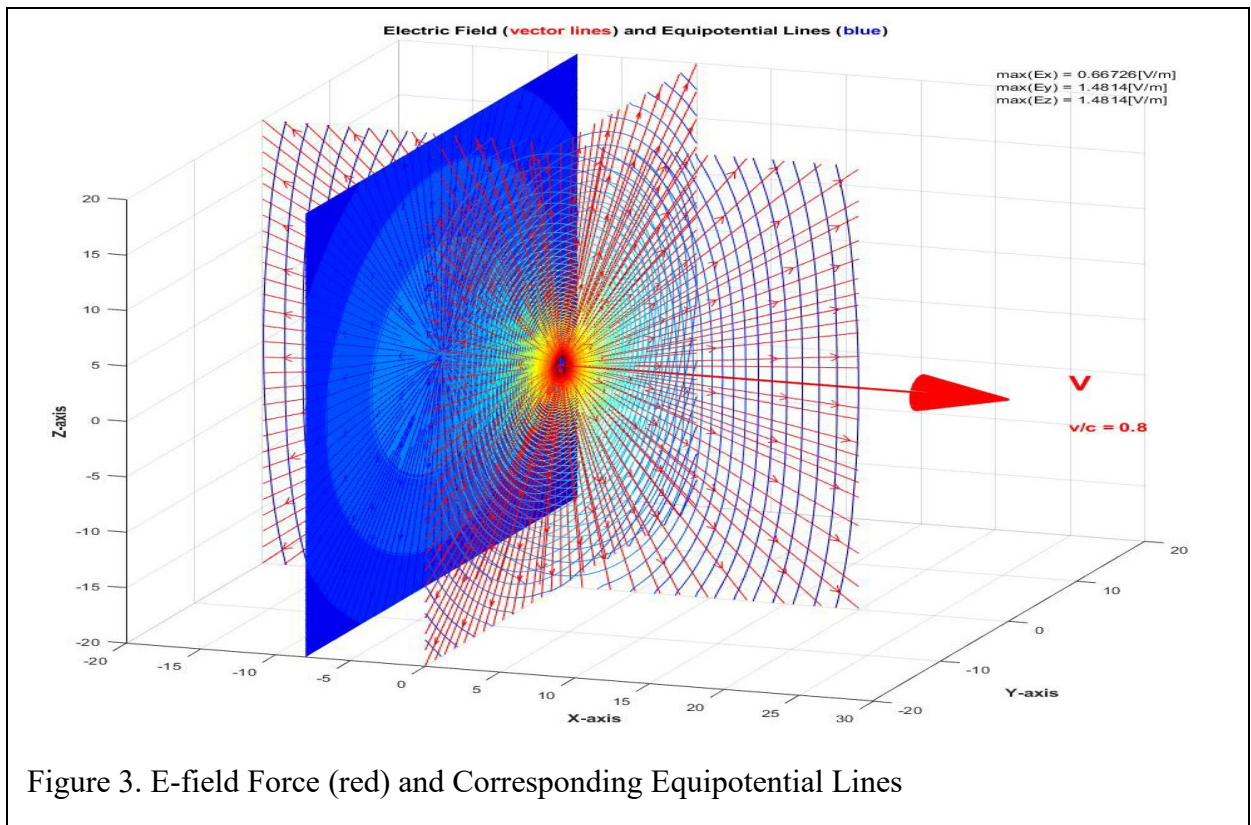


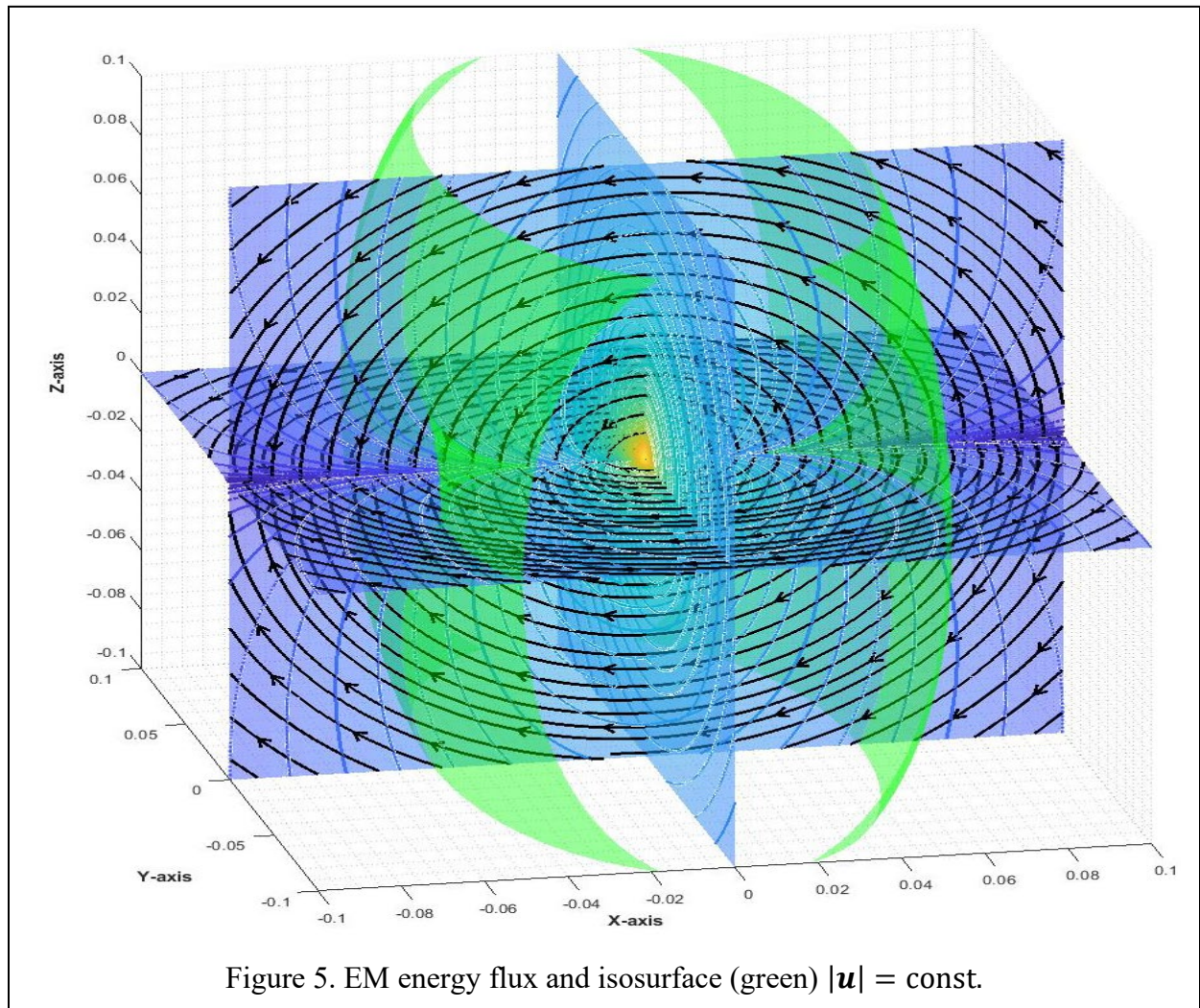
Figure 2. Isosurface of scalar (left) and vector (right) potential

The next two screenshots in Figures 3 and 4 display the animated E- and B-field distribution and their force lines (red in Figure 4 and blue in Figure 5) in the cardinal cross-sections. The vertical ZY-plane moves along the x -axis picturing the E- and B-field intensity and vectorial structure. The color bar (not shown in figures) is calibrated in the natural logarithm of E- or B-field energy proportional to the field magnitude square.

⁵R. Fitzpatrick, *Classical Electromagnetism*, The University of Texas at Austin, Section: *The electromagnetic energy tensor*, <https://farside.ph.utexas.edu/teaching/em/lectures/lectures.html>



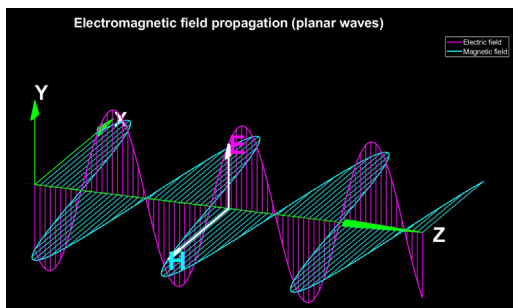
The last Figure 5 demonstrates the EM energy flux and the shape of its isosurface (green) $|u| = \text{const.}$



Poynting's vector energy trajectories are remarkable because all of them (black arrows) are winding around the charge and closed, i.e., the number of inbound and outbound E-energy force lines is equal for any closed surface with the charge inside. The number of force lines increases in areas with E-field concentration in the vicinity of the charge. It means the EM energy moves with the charge synchronously without radiation².

Go to <https://1drv.ms/f/s!AjtsKS-uvNP1avsbUFY9UsxLVVE>, click subsequently on the icon Document and MovingCharge. Mark, copy and download MovingChargeFile.m file to a newly created directory named Moving Charge, for example. Then open the MovingChargeFile.m file in Matlab and run the animation.

Problem 6. Linear Polarized Plane Wave Propagation in Free Space. Refresh the material of Sections



4.1.8 in Chapter 4. Go to <https://1drv.ms/f/s!AjtsKS-uvNP1avsbUFY9UsxLVVE>, click subsequently on the icon Document and PlaneWaveAnimation. Mark, copy and download PlanedWave.m file to a newly created directory named Plane Wave, for example. Then open the PlanedWave.m file in Matlab and run the animation. Pay attention to the E- and H-field reciprocal orientation and their variations in time and space. *How is Poynting's vector directed at any moment in time?*

